Mathematics 2

Mathematics Department
Phillips Exeter Academy
Exeter, NH
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To the Student

Contents: Members of the PEA Mathematics Department have written these materials. As you work through the problems, you will discover that algebra, geometry, and trigonometry have been integrated into a mathematical whole. Unlike textbooks you may have used in the past, there are no chapters or sections on specific topics. The whole curriculum is problem-centered, rather than topic-centered. Techniques and theorems become apparent as you work through the problems, and we encourage you to keep a notebook of the work you have done along with your notes from class discussions. These materials are designed to give you the tools for self-discovery. The first use of a key word is italicized and defined in the Reference.

Problem solving: We encourage you to approach each problem as an exploration. Read each question carefully! It is important to make accurate diagrams. Here are a few other useful strategies to keep in mind: make and solve an easier problem first, try a guess-and-check technique, recall work on a similar problem. It is important that you work on each problem when assigned, since any work you bring to class can help you and your class understand and uncover a solution. Problem solving requires persistence as much as it requires ingenuity. When you get stuck, or solve a problem incorrectly, back up and start over. Keep in mind that you're probably not the only one who is stuck. Bring to class a written record of your efforts, not just a blank space in your notebook. The methods that you use to solve a problem, the corrections that you make in your approach, the means by which you test the validity of your solutions, and your ability to communicate ideas are just as important as getting the correct answer.

Technology: Most of the problems in this book can be done without technology (graphing calculators, computer software, or applications). Nevertheless, you are encouraged to thoughtfully use technology to explore, and to formulate and test conjectures. It is important to use technology wisely. The technology that you have access to for homework may not be what is allowed on a test/quiz. Keep the following guidelines in mind: *think before you reach for a device*; keep notes in your notebook, so that you will have a clear record of what you have done; be wary of rounding mid-calculation; pay attention to the degree of accuracy requested; and be prepared to explain your method to your classmates. If you are asked to graph a curve the expectation is that, although you might use a graphing tool to generate a picture of the curve, you should sketch that picture in your notebook, with correctly scaled axes. If you do not know how to perform a needed action, there are many resources available online, but *beware of substituting online research for learning through self-discovery.*

Standardized testing: Standardized tests like the SAT, ACT, and Advanced Placement tests require calculators for certain problems. Although the Mathematics Department promotes the use of a variety of tools, it is still useful for students to know how to use hand-held graphing calculators. While many of the math faculty are well-versed in using these calculators and can answer questions as they arise, we leave it to students to determine the appropriate device for their test and manage their own practice with their device.

A Guide for New Students (For students, by students!)

Annually, approximately 320 new students enroll in a Mathematics course at PEA, and students arrive here from all over the world. As a new student, you will quickly come to realize the distinct methods and philosophies of teaching at Exeter. One aspect of Exeter that often catches students unaware is the math curriculum. We encourage all new students to come to the math table with a clear mind. You may not grasp, understand, or even like math at first, but you will have to be prepared for anything that comes before you.

During the fall of 2000, some new students voiced a concern about the math curriculum. The concerns ranged from grading, to math policies, and even to the very different teaching styles utilized in the mathematics department. This guide was originally written by students and has been updated by faculty to reflect changes since 2000, with the intent of preparing you for the task that you have embarked upon. It includes tips for survival and aspects of math unique to Exeter. Hopefully, this guide will ease your transition into math at Exeter. Remember,"Anything worth doing, is hard to do." —Mr. Higgins '36.

- *"I learned a lot more by teaching myself than by being taught by someone else."*
- *"One learns many ways to do different problems. Since each problem is different, you are forced to use all aspects of math."*
- *"It makes me think more. The way the math books are setup (i.e. simple problems progressing to harder ones on a concept) really helps me understand the mathematical concepts."*
- *"When you discover or formulate a concept yourself, you remember it better and understand the concept better than if we memorized it or the teacher just told us that the formula was 'xyz'."*

Homework

Math homework = no explanations and roughly eight problems a night. For the most part, it has become standard among most math teachers to give about eight problems a night; but I have even had a teacher who gave ten — though two problems may not seem like a big deal, it can be. Since all the problems are scenarios, and often have topics that vary, they also range in complexity, from a simple, one-sentence question, to a full-fledged paragraph with an eight-part answer! Don't fret though, transition to homework will come with time, similar to how you gain wisdom, as you get older. Homework can vary greatly from night to night, so try to be flexible with your time. With the large workload Exonians carry, no teacher should ever expect you to spend more time than what is allotted by the school's homework guidelines. Try your hardest to concentrate and utilize that time efficiently, avoiding distractions.

Without any explanations showing you exactly how to do your homework, how are you supposed to do a problem that you have absolutely no clue about? (This WILL happen!) The problems build on one another, so reviewing your notes from your previous class meeting is a good place to start. You could also reach out to a classmate or someone in your dorm. Another person in your dorm might be in the same class, or the same level, or possibly in a higher level of math. Remember, there is a difference between homework and studying. After you're through with the problems assigned, it is good practice to go back over your work from the last few days.

Going to the Board

It is very important to go to the board to put up homework problems. Usually, every homework problem is put up on the board at the beginning of class, and then they are discussed in class. If you regularly put problems up on the board, your teacher will have a good feel of where you stand in the class; a confident student will most likely be more active in participating in the class.

Plagiarism

One thing to keep in mind is plagiarism. You can get help from almost anywhere, but make sure that you cite your help, and that all work shown or turned in is your own, even if someone else showed you how to do it. Teachers do occasionally give problems/quizzes/tests to be completed at home. You may not receive help on these assessments, unless instructed to by your teacher; it is imperative that all the work is yours. When in doubt, ask your teacher.

Math Help

Getting help is an integral part of staying on top of the math program here at Exeter. It can be rather frustrating to be lost and feel you have nowhere to turn. There are a few tricks of the trade however, which ensure your "safety," with this possibly overwhelming word problem extravaganza.

Collaboration on homework is often useful. If you find that you are struggling, it may help to work on homework alongside a classmate so that when questions arise you can help one another. You could also reach out to some other friend who might be in the same class, or the same level, or possibly in a higher level of math. The Learning Center offers nightly math help. The Learning Center hours may overlap with your dorm check-in time. If so, be sure to ask the faculty member on duty in advance for permission to check out to the Learning Center for math help. You can always ask your math teacher for help. If there is no time during the day, it may be possible to check out of the dorm after your check-in time, to meet with your teacher. It is easiest to do this on the nights that your teacher is on duty in his/her dorm.

Math Help at the Learning Center

From 7:00 - 9:30 PM Sunday-Thursday, math help is available in the Learning Center. Starting in Fall 2021, a math department faculty member is on duty in the Learning Center along with Peer Tutors. Different people are available each night and some nights and hours are busier than others, so it is worth trying to visit at different times to determine what works for you.

Different Teachers Teach Differently

The teachers at Exeter usually develop their own style of teaching, fitted to their philosophy of the subject they teach; it is no different in the math department. Teachers vary at all levels: they grade differently, assess your knowledge differently, teach differently, and go over homework differently. They offer help differently, too. This simply means that it is essential that you be prepared each term to adapt to a particular teaching style. For instance, my teacher tests me about every two weeks, gives hand-in problems every couple of days, and also gives a few quizzes. However, my friends, who are in the same level math as I am, have teachers that in one case just gives weekly quizzes while the other teacher bases their grade on just two tests. Don't be afraid to ask your teacher how they grade. You must learn to be flexible to teaching styles and even your teacher's personality. This is a necessity for all departments at Exeter, including math.

- *"The tests are the hardest part between terms to adapt to, but if you prepare well, there shouldn't be a problem."*
- *"My other teacher taught and pointed out which problems are related when they are six pages apart."*
- *"It took a few days adjusting to, but if you pay attention to what the teacher says and ask him/her questions about their expectations, transitions should be smooth."*
- *"Every teacher gave different amounts of homework and tests. Class work varied too. My fall term teacher made us put every problem on the board, whereas my winter term teacher only concentrated on a few."*

New Student Testimonials

When I started this term in math, I was worried that the class would be easy. I had just finished an extensive course on Algebra 2, and I felt like I was taking a step back into geometry. Over the course of the past two and a half months, I was proven very wrong.

This course ended up being challenging and thought-provoking, and I feel like I was pushed to my mathematical limits and strengthened because of it. Not only were the tests difficult at times, but I also had to find my voice in Harkness and presentations. I ended up realizing just how much geometry I had not learned, and I now feel so much more confident in my math abilities. I'm excited for when next term starts because I'll have a clean slate for math, and I feel like I now understand what I need to do and how I can further improve on my work. Having those couple of 86 tests made me realize that I needed to work more without dropping my grade past a point of no return.

I found parametrics in 3D to be a bit confusing, but I was interested in learning about their relationship with the world (ex. will these two objects hit each other, or will their paths intersect). Something that I enjoyed about the class was the build-up to realizations, like how you showed us that if something comes into a parabolic radar dish, then it will bounce off into the focus.

All in all, I was challenged in this class, but I feel like it helped me understand what I need to do to succeed at Exeter. I'm much more confident now in my ability to do well, and I am glad that I was able to start off my time here with a (fairly) successful term in math.

— *New Lower, Math 23x, '26*

After eight years of math textbooks and lecture-style math classes, math at Exeter was a lot to get used to. My entire elementary math education was based on reading how to do problems from the textbook, then practicing monotonous problems that had no real-life relevance, one after the other. This method is fine for some people, but it wasn't for me. By the time I came to Exeter, I was ready for a change of pace, and I certainly got one.

Having somewhat of a background in algebra, I thought the Transition 1 course was just right for me. It went over basic algebra and problem-solving techniques. The math books at Exeter are very different from traditional books. They are compiled by the teachers, and consist of pages upon pages of word problems that lead you to find your own methods of solving problems. The problems are not very instructional, they lay the information down for you, most times introducing new vocabulary, (there is an index in the back of the book), and allow you to think about the problem, and solve it any way that you can. When I first used this booklet, I was a little thrown back; it was so different from everything I had done before — but by the time the term was over, I had the new method down.

The actual math classes at Exeter were hard to get used to as well. Teachers usually assign about eight problems a night, leaving you time to "explore" the problems and give each one some thought. Then, next class, students put all the homework problems on the board. The class goes over each problem; everyone shares their method and even difficulties that they ran into while solving it. I think the hardest thing to get used to, is being able to openly ask questions. No one wants to be wrong, I guess it is human nature, but in the world of Exeter math, you can't be afraid to ask questions. You have to seize the opportunity to speak up and say "I don't understand," or "How did you get that answer?" If you don't ask questions, you will never get the answers you need to thrive.

Something that my current math teacher always says is to make all your mistakes on the board, because when a test comes around, you don't want to make mistakes on paper. This is so true, class time is practice time, and it's hard to get used to not feeling embarrassed after you answer problems incorrectly. You need to go out on a limb and try your best. If you get a problem wrong on the board, it's one new thing learned in class, not to mention, one less thing to worry about messing up on, on the next test.

Math at Exeter is really based on cooperation, you, your classmates, and your teacher. It takes a while to get used to, but in the end, it is worth the effort. — *Hazel Cipolle '04*

I entered my second math class of Fall Term as a ninth grader, with a feeling of dread. Though I had understood the homework the night before, I looked down at my paper with a blank mind, unsure how I had done any of the problems. The class sat nervously around the table until we were prompted by the teacher to put the homework on the board. One boy stood up and picked up some chalk. Soon others followed suit. I stayed glued to my seat with the same question running through my mind, what if I get it wrong?

I was convinced that everyone would make fun of me, that they would tear my work apart, that each person around that table was smarter than I was. I soon found that I was the only one still seated and hurried to the board. The only available problem was one I was slightly unsure of. I wrote my work quickly and reclaimed my seat.

We reviewed the different problems, and everyone was successful. I explained my work and awaited the class' response. My classmates agreed with the bulk of my work, though there was a question on one part. They suggested different ways to find the answer and we were able to work through the problem, together.

I returned to my seat feeling much more confident. Not only were my questions cleared up, but my classmates' questions were answered as well. Everyone benefited.

I learned one of the more important lessons about math at Exeter that day; it doesn't matter if you are right or wrong. Your classmates will be supportive of you, and be tolerant of your questions. Chances are, if you had trouble with a problem, someone else in the class did too. Another thing to keep in mind is that the teacher expects nothing more than that you try to do a problem to the best of your ability. If you explain a problem that turns out to be incorrect, the teacher will not judge you harshly. They understand that no one is always correct, and will not be angry or upset with you. — *Elisabeth Ramsey '04*

I never thought math would be a problem. That is, until I came to Exeter. I entered into Math 12T, clueless as to what the curriculum would be. The day I bought the Math 1 book from the bookstore, I stared at the problems in disbelief. ALL WORD PROBLEMS. "Why word problems?" I thought. I had dreaded word problems ever since I was a second grader, and on my comments it always read, "Charly is a good math student, but she needs to work on word problems." I was in shock. I would have to learn math in an entirely new language. I began to dread my B-format math class.

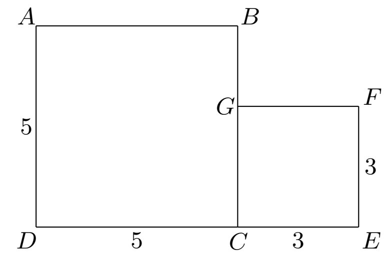
My first math test at Exeter was horrible. I had never seen a D- on a math test. Never. I was upset and I felt dumb, especially since others in my class got better grades, and because my roommate was extremely good in math. I cried. I said I wanted to go home where things were easier. But finally I realized, "I was being given a challenge. I had to at least try."

I went to my math teacher for extra help. I asked questions more often (though not as much as I should have), and slowly I began to understand the problems better. My grades gradually got better, by going from a D- to a C+ to a B and eventually I got an A-. It was hard, but that is Exeter. You just have to get passed that first hump, though little ones will follow. As long as you don't compare yourself to others, and you ask for help when you need it, you should get used to the math curriculum. I still struggle, but as long as I don't get intimidated and don't give up, I am able to bring my grades up. — *Charly Simpson '04*

- *"At first, I was very shy and had a hard time asking questions."*
- *"Sometimes other students didn't explain problems clearly."*
- *"Solutions to certain problems by other students are sometimes not the fastest or easiest. Some students might know tricks and special techniques that aren't covered."*
- *"My background in math was a little weaker than most people's, therefore I was unsure how to do many of the problems. I never thoroughly understood how to do a problem before I saw it in the book."*

Mathematics 2

1. A 5×5 square and a 3×3 square can be cut into pieces that will fit together to form a third square.
(a) Find the length of a side of the third square.
(b) In the diagram at right, mark P on *segment* DC so that $PD = 3$, then draw segments PA and PF. Calculate the lengths of these segments.
(c) Segments PA and PF divide the squares into pieces. Arrange the pieces to form the third square.



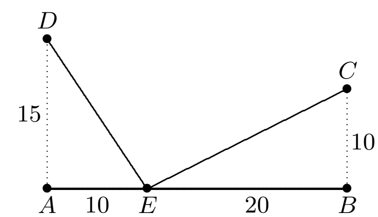
2. (Continuation) Change the sizes of the squares to $AD = 8$ and $EF = 4$, and redraw the diagram. Where should point P be marked this time? Form the third square again.

3. (Continuation) Will the preceding method *always* produce pieces that form a new square? If your answer is *yes*, prepare a written explanation. If your answer is *no*, provide a counterexample — two specific squares that can *not* be converted to a single square.

4. Instead of walking along two sides of a rectangular field, Fran took a shortcut along the diagonal, thus saving distance equal to half the length of the longer side. Find the length of the long side of the field, given that the the length of the short side is 156 meters.

5. Let $A = (0, 0)$, $B = (7, 1)$, $C = (12, 6)$, and $D = (5, 5)$. Plot these points and connect the dots to form the quadrilateral $ABCD$. Verify that all four sides have the same length. Such a figure is called *equilateral*.

6. Two iron rails, each 50 feet long, are laid end to end with no space between them. During the summer, the heat causes each rail to increase in length by 0.04 percent. Although this is a small increase, the lack of space at the joint makes the joint buckle upward. What distance upward will the joint be forced to rise? [Assume that each rail *remains straight*, and that the other ends of the rails are anchored.]



7. In the diagram, AEB is straight and angles A and B are right. Calculate the total distance $DE + EC$.

8. (Continuation) If $AE = 20$ and $EB = 10$ instead, would $DE + EC$ be the same?

9. (Continuation) You have seen that the value chosen for AE determines the value of $DE + EC$. One also says that $DE + EC$ is a function of AE. Letting x stand for AE (and $30 - x$ for EB), write a formula for this function. Graph this formula using a graphing tool. Locate the point on the graph that represents the *shortest* path from D to C through E. Draw an accurate picture of this path, and make a conjecture about angles AED and BEC. Consider the slopes or use a protractor to test your conjecture.

10. Two different points on the line $y = 2$ are each exactly 13 units from the point $(7, 14)$. Draw a picture of this situation, and then find the coordinates of these points.

11. Give an example of a point that is the same distance from $(3,0)$ as it is from $(7,0)$. Find lots of examples. Describe the configuration of all such points. In particular, how does this configuration relate to the two given points?

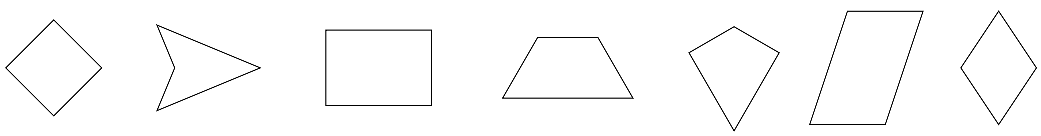
12. Verify that the hexagon formed by $A = (0,0)$, $B = (2,1)$, $C = (3,3)$, $D = (2,5)$, $E = (0,4)$, and $F = (-1,2)$ is equilateral. Is it also *equiangular*?

13. Draw a 20-by-20 square $ABCD$. Mark P on AB so that $AP = 8$, Q on BC so that $BQ = 5$, R on CD so that $CR = 8$, and S on DA so that $DS = 5$. Find the lengths of the sides of quadrilateral $PQRS$. Is there anything special about this quadrilateral? Explain.

14. Verify that $P = (1,-1)$ is the same distance from $A = (5,1)$ as it is from $B = (-1,3)$. It is customary to say that P is *equidistant* from A and B. Find three more points that are equidistant from A and B. By the way, to "find" a point means to find its *coordinates*. Can points equidistant from A and B be found in every *quadrant*?

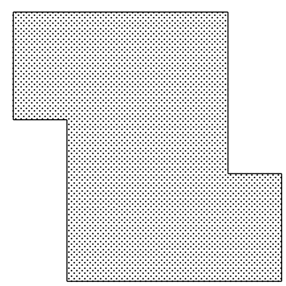
15. Inside a 5-by-5 square, it is possible to place four 3-4-5 triangles so that they do not overlap. Show how. Then explain why you can be sure that it is impossible to squeeze in a fifth triangle of the same size.

16. Which of the following figures would you describe as a *kite*? Explain your reasoning.



17. Find both points on the line $y = 3$ that are 10 units from $(2,-3)$.

18. On a number line, where is $\frac{1}{2}(p+q)$ in relation to p and q?



19. Some terminology: Figures that have exactly the same shape and size are called *congruent*. Dissect the region shown at right into two congruent parts. How many different ways of doing this can you find?

20. Let $A = (2,4)$, $B = (4,5)$, $C = (6,1)$, $T = (7,3)$, $U = (9,4)$, and $V = (11,0)$. Triangles ABC and TUV are specially related to each other. Make calculations to clarify this statement, and write a few words to describe what you discover.

21. A triangle that has two sides of equal length is called *isosceles*. Make up an example of an isosceles triangle, one of whose *vertices* is $(3,5)$. If you can, find a triangle that does not have any horizontal or vertical sides.

22. Una recently purchased two boxes of ten-inch candles — one box from a discount store, and the other from an expensive boutique. It so happens that the inexpensive candles last only three hours each, while the expensive candles last five hours each. One evening, Una hosted a dinner party and lighted two candles — one from each box — at 7:30 pm. During dessert, a guest noticed that one candle was twice as long as the other. At what time was this observation made?

23. Let $A = (-4, 7)$ and $B = (2, -3)$. Verify that $P = (4, 5)$ is equidistant from A and B. Find at least two more points that are equidistant from A and B. Describe all such points.

24. Find two points on the y-axis that are 9 units from $(7, 5)$.

25. A *lattice point* is a point whose coordinates are *integers*. Find two lattice points that are exactly $\sqrt{13}$ units apart. Is it possible to find lattice points that are $\sqrt{15}$ units apart? Is it possible to form a square whose area is 18 by connecting four lattice points? Explain.

26. *Some terminology*: When two angles fit together to form a straight angle (a 180-degree angle, in other words), they are called *supplementary angles*, and either angle is the *supplement* of the other. When an angle is the same size as its supplement (a 90-degree angle), it is called a *right angle*. When two angles fit together to form a right angle, they are called complementary angles, and either angle is the *complement* of the other. Two lines that form a right angle are said to be *perpendicular*.

The three angles of a triangle fit together to form a straight angle. In one form or another, this statement is a fundamental *postulate* of Euclidean geometry — accepted as true, without proof. Taking this for granted, then, what can be said about the two non-right angles in a right triangle?

27. Let $P = (a, b)$, $Q = (0, 0)$, and $R = (-b, a)$, where a and b are positive numbers. Prove that angle PQR is right, by introducing two congruent right triangles into your diagram. Verify that the *slope* of *segment* QP is the *negative reciprocal* of the slope of segment QR.

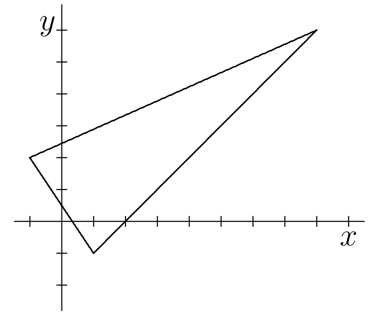
28. Find an example of an equilateral hexagon whose sides are all $\sqrt{13}$ units long. Give lattice-point coordinates for all six points.

29. I have been observing the motion of a bug that is crawling on my graph paper. When I started watching, it was at the point $(1, 2)$. Ten seconds later it was at $(3, 5)$. Another ten seconds later it was at $(5, 8)$. After another ten seconds it was at $(7, 11)$.
(a) Draw a picture that illustrates what is happening. What did you assume?
(b) Where was the bug 25 seconds after I started watching it? What did you assume?
(c) Where was the bug 26 seconds after I started watching it? What did you assume?

30. The point on segment AB that is equidistant from A and B is called the *midpoint* of AB. For each of the following, find coordinates for the midpoint of AB:
(a) $A = (-1, 5)$ and $B = (3, -7)$ **(b)** $A = (m, n)$ and $B = (k, l)$

31. Write a formula for the distance from $A = (-1, 5)$ to $P = (x, y)$, and another formula for the distance from $P = (x, y)$ to $B = (5, 2)$. Then write an equation that says that P is equidistant from A and B. Simplify your equation to *linear* form. This line is called the *perpendicular bisector of AB*. Verify this by calculating two slopes and one midpoint.

32. Find the slope of the line through
(a) $(3, 1)$ and $(3 + 4t, 1 + 3t)$, $t \neq 0$ **(b)** $(m - 5, n)$ and $(5 + m, n^2)$



33. Is it possible for a line $ax + by = c$ to lack a y-intercept? To lack an x-intercept? Explain.

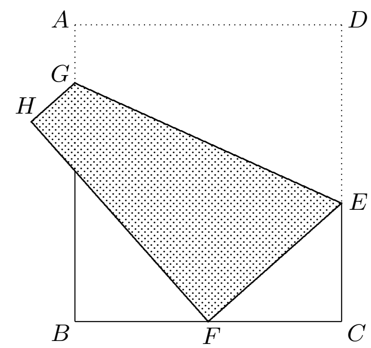
34. The sides of the triangle at right are formed by the graphs of $3x + 2y = 1$, $y = x - 2$, and $-4x + 9y = 22$. Is the triangle isosceles? How do you know?

35. Pat races at 10 miles per hour, while Kim races at 9 miles per hour. When they both ran in the same long-distance race last week, Pat finished 8 minutes ahead of Kim. What was the length of the race, in miles? Briefly describe your reasoning.

36. A bug moves linearly with constant speed across my graph paper. I first notice the bug when it is at $(3, 4)$. It reaches $(9, 8)$ after two seconds and $(15, 12)$ after four seconds.
(a) Predict the position of the bug after six seconds; after nine seconds; after t seconds.
(b) Is there a time when the bug is equidistant from the x- and y-axes? If so, where is it?

37. What is the relation between the lines described by the equations $-20x + 12y = 36$ and $-35x + 21y = 63$? Find a third equation in the form $ax + by = 90$ that fits this pattern.

38. Consider the linear equation $y = 3.62(x - 1.35) + 2.74$.
(a) What is the slope of this line?
(b) What is the value of y when $x = 1.35$?
(c) This equation is written in *point-slope* form. Explain the terminology.
(d) Use a graphing tool to graph this line.
(e) Find an equation for the line through $(4.23, -2.58)$ that is *parallel* to this line.
(f) Use point-slope form to write the equation of a line that has slope -1.25 and that goes through the point $(-3.75, 8.64)$.



39. The dimensions of rectangular piece of paper $ABCD$ are $AB = 10$ and $BC = 9$. It is folded so that corner D is matched with a point F on edge BC. Given that length $DE = 6$, find EF, EC, FC, and the area of EFC.

40. (Continuation) The lengths EF, EC, and FC are all functions of the length DE. The area of triangle EFC is also a function of DE. Using x to stand for DE, write formulas for these four functions.

41. (Continuation) Use a graphing tool to find the value of x that maximizes the area of triangle EFC.

42. The x- and y-coordinates of a point are given by the equations shown below. The position of the point depends on the value assigned to t. Use your graph paper to plot points corresponding to the values $t = -4, -3, -2, -1, 0, 1, 2, 3$, and 4. Do you recognize any patterns? Describe them.

$$\begin{cases} x = 2 + 2t \\ y = 5 - t \end{cases}$$

43. Plot the following points on the coordinate plane: $(1, 2)$, $(2, 5)$, $(3, 8)$. Write equations, similar to those in the preceding exercise, that produce these points when t-values are assigned. There is more than one correct answer.

44. Given that $2x - 3y = 17$ and $4x + 3y = 7$, find the value of x by hand.

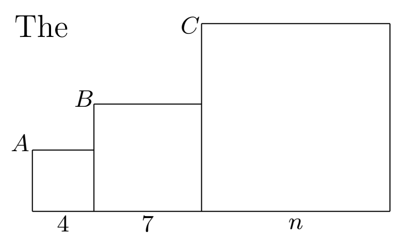
45. A slope can be considered to be a rate. Explain this interpretation.

46. Find a and b so that $ax + by = 1$ has x-intercept 5 and y-intercept 8.

47. Given points $A = (-2, 7)$ and $B = (3, 3)$, find two points P that are on the perpendicular bisector of AB. In each case, what can you say about segments PA and PB?

48. Explain the difference between a line that has *no slope* and a line whose slope is *zero*.

49. Three squares are placed next to each other as shown. The vertices A, B, and C are *collinear*. Find the dimension n.



50. (Continuation) Replace the lengths 4 and 7 by m and k, respectively. Express k in terms of m and n.

51. A five-foot *prep* casts a shadow that is 40 feet long while standing 200 feet from a streetlight. How high above the ground is the lamp?

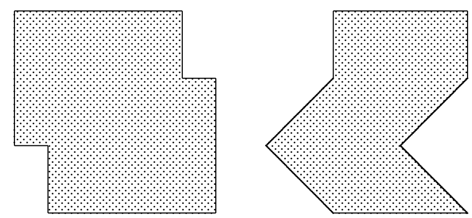
52. (Continuation) How far from the streetlight should the prep stand, in order to cast a shadow that is exactly as long as the prep is tall?

53. An airplane 27 000 feet above the ground begins descending at the rate of 1500 feet per minute. Assuming the plane continues at the same rate of descent, how long will it be before it is on the ground?

54. (Continuation) Using an appropriate window, graph the line $y = 27\,000 - 1500x$. With the preceding problem in mind, explain the significance of the slope of this line and its two intercepts.

55. An airplane is flying at 36000 feet directly above Lincoln, Nebraska. A little later the plane is flying at 28000 feet directly above Des Moines, Iowa, which is 160 miles from Lincoln. Assuming a constant rate of descent, predict how far from Des Moines the airplane will be when it lands.

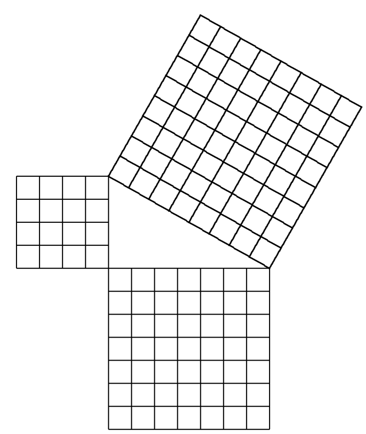
56. In a dream, Blair is confined to a coordinate plane, moving along a line with a constant speed. Blair's position at 4 am is $(2, 5)$ and at 6 am it is $(6, 3)$. What is Blair's position at 8:15 am when the alarm goes off?



57. Find a way to show that points $A = (-4, -1)$, $B = (4, 3)$, and $C = (8, 5)$ are collinear.

58. Find as many ways as you can to dissect each figure at right into two congruent parts.

59. Let $A = (4, 2)$, $B = (11, 6)$, $C = (7, 13)$, and $D = (0, 9)$. Show that $ABCD$ is a square.



60. Is there anything wrong with the figure shown at right?

61. Show that a 9-by-16 rectangle can be transformed into a square by dissection. In other words, the rectangle can be cut into pieces that can be reassembled to form the square. Do it with as few pieces as possible.

62. At noon one day, Corey decided to follow a straight course in a motor boat. After one hour of making no turns and traveling at a steady rate, the boat was 6 miles east and 8 miles north of its point of departure. What was Corey's position at two o'clock? How far had Corey traveled? What was Corey's speed?

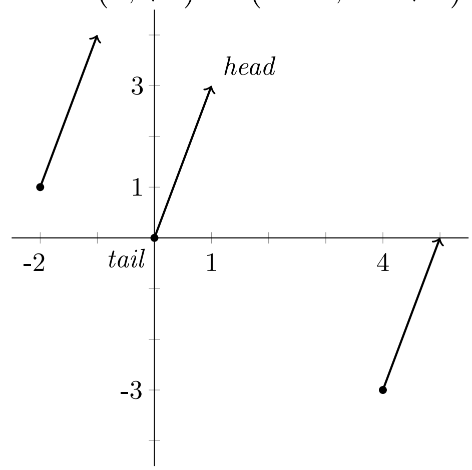
63. (Continuation) Assume that the fuel tank initially held 12 gallons, and that the boat gets 4 miles to the gallon. How far did Corey get before running out of fuel? When did this happen? When radioing the Coast Guard for help, how should Corey describe the boat's position?

64. Suppose that numbers a, b, and c fit the equation $a^2 + b^2 = c^2$, with $a = b$. Express c in terms of a. Draw a good picture of such a triangle. What can be said about its angles?

65. The Krakow airport is 3 km west and 5 km north of the city center. At 1 pm, Zuza took off in a Cessna 730. Every six minutes, the plane's position changed by 9 km east and 7 km north. At 2:30 pm, Zuza was flying over the town of Jozefow. In relation to the center of Krakow, **(a)** where is Jozefow? **(b)** where was Zuza after t hours of flying?

66. Draw the following segments. What do they have in common?
from $(3, -1)$ to $(4, 2)$; from $(1.3, 0.8)$ to $(2.3, 3.8)$; from $\left(\pi, \sqrt{2}\right)$ to $\left(1 + \pi, 3 + \sqrt{2}\right)$.

67. (Continuation) The *directed segments* have the same *length* and the same direction. Each represents the *vector* $[1, 3]$. The *components* of the vector are the numbers 1 and 3. The initial point of the segment is called the *tail* of the vector, and the final point is called the *head*. This is labeled for one of the vectors to the right.
(a) Label the heads and tails of the other vectors and find plausible coordinates for them.
(b) Which of the following directed segments represents $[7, 4]$? from $(-2, -3)$ to $(5, -1)$; from $(-3, -2)$ to $(11, 6)$; from $(10, 5)$ to $(3, 1)$; from $(-7, -4)$ to $(0, 0)$.



68. Points (x, y) described by the equations $x = 1 + 2t$ and $y = 3 + t$ form a line. Is the point $(7, 6)$ on this line? How about $(-3, 1)$? How about $(6, 5.5)$? How about $(11, 7)$?

69. The perimeter of an isosceles right triangle is 24 cm. How long are its sides?

70. The x- and y-coordinates of points that depend on t are given by the equations shown below. Use your graph paper to plot points corresponding to $t = -1$, 0, and 2. These points should appear to be collinear. Convince yourself that this is the case, and calculate the slope of this line. The displayed equations are called *parametric*, and t is called a *parameter*. How is the slope of a line determined from its parametric equations?

$$\begin{cases} x = -4 + 3t \\ y = 1 + 2t \end{cases}$$

71. Find parametric equations to describe the line that goes through the points $A = (5, -3)$ and $B = (7, 1)$. There is more than one correct answer to this question.

72. Show that the triangle formed by the lines $2x - y = 7$, $x + 2y = 16$, and $3x + y = 13$ is isosceles. Show also that the lengths of the sides of this triangle fit the Pythagorean equation. Can you identify the right angle just by looking at the equations?

73. Leaving home on a recent business trip, Kyle drove 10 miles south to reach the airport, then boarded a plane that flew a straight course — 6 miles east and 3 miles north each minute. What was the airspeed of the plane? After two minutes of flight, Kyle was directly above the town of Greenup. How far is Greenup from Kyle's home? A little later, the plane flew over Kyle's birthplace, which is 50 miles from home. When did this occur?

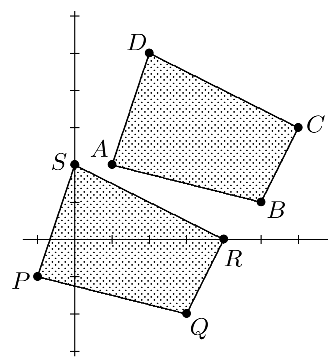
74. A triangle has vertices $A = (1, 2)$, $B = (3, -5)$, and $C = (6, 1)$. *Image triangle* $A'B'C'$ is obtained by *sliding* triangle ABC 5 units to the right (in the positive x-direction, in other words) and 3 units up (in the positive y-direction). It is also customary to say that vector $[5, 3]$ has been used to *translate* triangle ABC. What are the coordinates of *image points* A', B', and C'? By the way, "C prime" is the usual way of reading C'.

75. (Continuation) When vector $[h, k]$ is used to translate triangle ABC, it is found that the image of *vertex* A is $(-3, 7)$. What are the images of vertices B and C?

76. In a dream, Blair is moving along the line $y = 3x + 2$. At midnight, Blair's position is $(1, 5)$, the x-coordinate increasing by 4 units every hour. Write parametric equations that describe Blair's position t hours after midnight. What was Blair's position at 10:15 pm when the dream started? Find Blair's speed, in units per hour.

77. The parametric equations $x = -2 - 3t$ and $y = 6 + 4t$ describe the position of a particle, in meters and seconds. How does the particle's position change each second? each minute? What is the speed of the particle, in meters per second? Write parametric equations that describe the particle's position, using meters and *minutes* as units.

78. Let $A = (1, 2)$, $B = (5, 1)$, $C = (6, 3)$, and $D = (2, 5)$. Let $P = (-1, -1)$. $Q = (3, -2)$, $R = (4, 0)$, and $S = (0, 2)$. Use a vector to describe how quadrilateral $ABCD$ is related to quadrilateral $PQRS$.



79. Let $K = (3, 8)$, $L = (7, 5)$, and $M = (4, 1)$. Find coordinates for the vertices of the triangle that is obtained by using the vector $[2, -5]$ to slide triangle KLM. How *far* does each vertex slide?

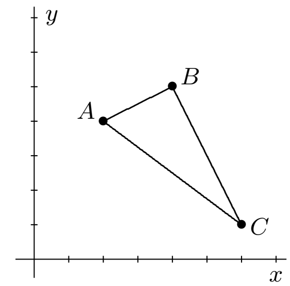
80. Find parametric equations that describe the following lines:
(a) through $(3, 1)$ and $(7, 3)$ **(b)** through $(7, -1)$ and $(7, 3)$

81. Find all points on the y-axis that are twice as far from $(-5, 0)$ as they are from $(1, 0)$. Begin by making a drawing and estimating. Find all such points on the x-axis. In each case, how many points did you find? How do you know that you have found them all?

82. Let $A = (-5, 0)$, $B = (5, 0)$, and $C = (2, 6)$; let $K = (5, -2)$, $L = (13, 4)$, and $M = (7, 7)$. Verify that the length of each side of triangle ABC matches the length of a side of triangle KLM. Because of this data, it is natural to regard the triangles as being in some sense equivalent. It is customary to call the triangles *congruent*. The basis used for this judgment is called the *side-side-side* criterion. What can you say about the sizes of angles ACB and KML? What is your reasoning? What about the other angles?

83. (Continuation) Are the triangles related by a vector translation? Why or why not?

84. Let $A = (2, 4)$, $B = (4, 5)$, and $C = (6, 1)$. Triangle ABC is shown at right. Draw three new triangles as follows:
(a) ΔPQR has $P = (11, 1)$, $Q = (10, -1)$, and $R = (6, 1)$;
(b) ΔKLM has $K = (8, 10)$, $L = (7, 8)$, and $M = (11, 6)$;
(c) ΔTUV has $T = (-2, 6)$, $U = (0, 5)$, and $V = (2, 9)$.
These triangles are not obtained from ABC by applying vector translations. Instead, each of the appropriate *transformations* is described by one of the suggestive names *reflection*, *rotation*, or *glide-reflection*. Decide which is which, with justification.



85. A bug is moving along the line $3x + 4y = 12$ with constant speed 5 units per second. The bug crosses the x-axis when $t = 0$ seconds. It crosses the y-axis later. When? Where is the bug when $t = 2$? when $t = -1$? when $t = 1.5$? What does a negative t-value mean?

86. Give the components of the vector whose length is 10 and whose direction *opposes* the direction of $[-4, 3]$.

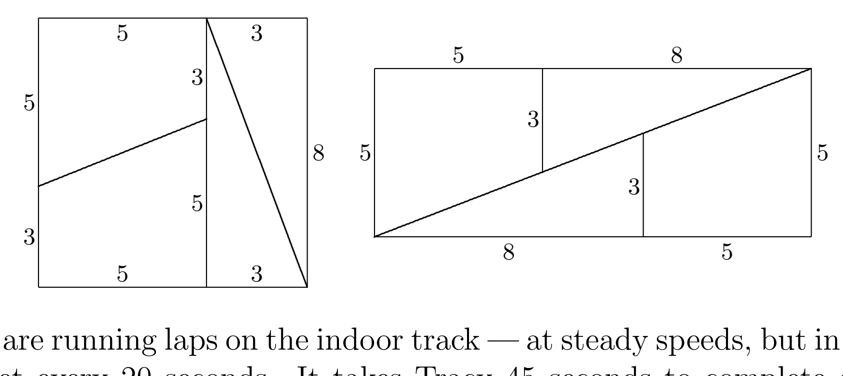
87. Find the coordinates for the point that is three fifths of the way from $(4, 0)$ to $(0, 3)$. Start by writing parametric equations to describe the line joining those two points.

88. Let $A = (0, 0)$, $B = (2, -1)$, $C = (-1, 3)$, $P = (8, 2)$, $Q = (10, 3)$, and $R = (5, 3)$. Plot these points. Angles BAC and QPR should look like they are the same size. Find evidence to support this conclusion.

89. An equilateral quadrilateral is called a *rhombus*. A square is a simple example of a rhombus. Find a non-square rhombus whose *diagonals* and sides are not parallel to the rulings on your graph paper. Use coordinates to describe its vertices. Write a brief description of the process you used to find your example.

90. Using a ruler and protractor, draw a triangle that has an 8-cm side and a 6-cm side, which make a 30-degree angle. This is a *side-angle-side* (SAS) description. Cut out the figure so that you can compare triangles with your classmates. Will your triangles be congruent?

91. Compare the two figures shown below. Is there anything wrong with what you see?



92. Tracy and Kelly are running laps on the indoor track — at steady speeds, but in opposite directions. They meet every 20 seconds. It takes Tracy 45 seconds to complete each lap. How many seconds does it take for each of Kelly's laps? Check your answer.

93. Instead of saying that Remy moves 3 units left and 2 units up, you can say that Remy's position is displaced by the vector $[-3, 2]$. Identify the following *displacement vectors*:
(a) Forrest starts at $(2, 3)$ at 1 pm, and has moved to $(5, 9)$ by 6 am the next morning;
(b) at noon, Eugene is at $(3, 4)$; two hours earlier Eugene was at $(6, 2)$;
(c) during a single hour, a small airplane flew 40 miles north and 100 miles west.

94. Kirby moves with constant speed 5 units per hour along the line $y = \frac{3}{4}x + 6$, crossing the y-axis at midnight and the x-axis later.
(a) When is the x-axis crossing made?
(b) What does it mean to say that Kirby's position is a *function* of time?
(c) What is Kirby's position 1.5 hours after midnight?
(d) What is Kirby's position t hours after midnight?

95. A bug is initially at $(-3, 7)$. Where is the bug after being displaced by vector $[-7, 8]$?

96. With the aid of a ruler and protractor, draw a triangle that has an 8-cm side, a 6-cm side, and a 45-degree angle that is not formed by the two given sides. This is a *side-side-angle* (SSA) description. Cut out the figure so that you can compare triangles with your classmates. Do you expect your triangles to be congruent?

97. Plot points $K = (0, 0)$, $L = (7, -1)$, $M = (9, 3)$, $P = (6, 7)$, $Q = (10, 5)$, and $R = (1, 2)$. Show that the triangles KLM and RPQ are congruent. Show also that neither triangle is a vector translation of the other. Describe how one triangle has been transformed into the other.

98. (Continuation) If two figures are congruent, then their parts *correspond*. In other words, each part of one figure has been matched with a definite part of the other figure. In the triangle PQR, which angle corresponds to angle M? Which side corresponds to KL? In general, what can be said about corresponding parts of congruent figures? How might you confirm your hunch experimentally?

99. What is the slope of the line $ax + by = c$? Find an equation for the line through the origin that is perpendicular to the line $ax + by = c$.

100. Choose a point P on the line $2x + 3y = 7$, and draw the vector $[2, 3]$ with its tail at P and its head at Q. Confirm that the vector is perpendicular to the line. What is the distance from Q to the line? Repeat the preceding, with a different choice for point P.

101. Let $A = (3, 2)$ and $B = (7, -10)$. What is the displacement vector that moves point A onto point B? What vector moves B onto A?

102. Let $M = (a, b)$, $N = (c, d)$, $M' = (a + h, b + k)$, and $N' = (c + h, d + k)$. Show that segments MN and $M'N'$ have the same length. Explain why this could be expected.

103. The position of a bug is described by the parametric equation $(x, y) = (2 - 12t, 1 + 5t)$, where distance is measured in cm and time is measured in seconds. Explain why the speed of the bug is 13 cm/sec. Change the equation to obtain the description of a bug moving along the same line with speed 26 cm/second.

104. Given the vector $[-5, 12]$, find the following vectors:
(a) same direction, twice as long **(b)** same direction, length 1
(c) opposite direction, length 10 **(d)** opposite direction, length c

105. *Some terminology*: When the components of the vector $[5, -7]$ are multiplied by a given number t, the result may be written either as $[5t, -7t]$ or as $t[5, -7]$. This is called the *scalar multiple* of vector $[5, -7]$ by the *scalar* t. Find components for the following scalar multiples:
(a) $[12, -3]$ by scalar 5 **(b)** $\left[\sqrt{5}, \sqrt{10}\,\right]$ by scalar $\sqrt{5}$
(c) $\left[-\frac{3}{4}, \frac{2}{3}\right]$ by scalar $-\frac{1}{2} + \frac{2}{6}$ **(d)** $[p, q]$ by scalar $\frac{p}{q}$

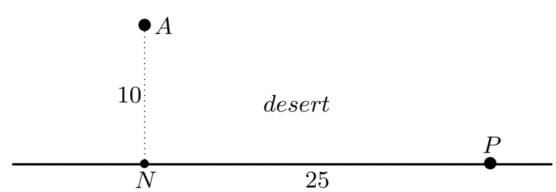
106. Find the lengths of the following vectors:
(a) $[3, 4]$ **(b)** $1998\,[3, 4]$ **(c)** $\frac{1998}{5}\,[3, 4]$ **(d)** $-2\,[3, 4]$ **(e)** $t[3, 4]$ **(f)** $t[a, b]$

107. With the aid of a ruler and protractor, cut out three non-congruent triangles, each of which has a 40-degree angle, a 60-degree angle, and an 8-cm side. One of your triangles has an *angle-side-angle* (ASA) description, while the other two have *angle-angle-side* (AAS) descriptions. What happens when you compare your triangles with those of your classmates?

108. (Continuation) If the parts of two triangles are matched so that two angles of one triangle are congruent to the corresponding angles of the other, and so that a side of one triangle is congruent to the corresponding side of the other, then the triangles must be congruent. Would the AAS criterion be a valid test for congruence if the word *corresponding* were left out of the definition? Explain.

109. The initial position of an object is $P_0 = (7, -2)$. Its position after being displaced by the vector $t[-8, 7]$ is $P_t = (7, -2) + t[-8, 7]$. Notice that the meaning of "+" is to apply a vector translation to P_0. Notice also that the position is a function of t. Calculate P_3, P_2, and P_{-2}. Describe the configuration of all possible positions P_t. By the way, P_t and P_2 are usually read "P sub t" and "P sub two".

110. Alex the geologist is in the desert, 10 km from a long, straight road. On the road, Alex's jeep can do 50 kph, but in the desert sands, it can manage only 30 kph. Alex is very thirsty, and knows that there is a gas station 25 km down the road (from the nearest point N on the road) that has ice-cold Pepsi.

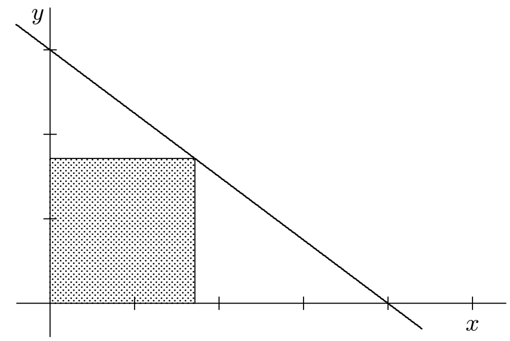


(a) How many minutes will it take for Alex to drive to P through the desert?
(b) Would it be faster if Alex first drove to N and then used the road to P?
(c) Find an even faster route for Alex to follow. Is your route the fastest possible?

111. Let $A = (1, 4)$, $B = (0, -9)$, $C = (7, 2)$, and $D = (6, 9)$. Prove that angles DAB and DCB are the same size. Can anything be said about the angles ABC and ADC?

112. Two of the sides of a right triangle have lengths $360\sqrt{2025}$ and $480\sqrt{2025}$. Find the possible lengths for the third side.

113. The diagram at right shows the graph of $3x + 4y = 12$. The shaded figure is a square, three of whose vertices are on the coordinate axes. The fourth vertex is on the line. Find
(a) the x- and y-intercepts of the line;
(b) the length of a side of the square.



114. (Continuation) Draw a rectangle that is twice as wide as it is tall, and that fits snugly into the triangular region formed by the line $3x + 4y = 12$ and the positive coordinate axes, with one corner at the origin and the opposite corner on the line. Find the dimensions of this rectangle.

115. Plot the three points $P = (1, 3)$, $Q = (5, 6)$, and $R = (11.4, 10.8)$. Verify that $PQ = 5$, $QR = 8$, and $PR = 13$. What is special about these points?

116. Sidney calculated three distances and reported them as $TU = 29$, $UV = 23$, and $TV = 54$. What do you think of Sidney's data, and why?

117. Find the number that is two thirds of the way **(a)** from -7 to 17; **(b)** from m to n.

118. The diagonal of a rectangle is 15 cm, and the perimeter is 38 cm. What is the area? It is possible to find the answer without finding the dimensions of the rectangle — try it.

119. After drawing the line $y = 2x - 1$ and marking the point $A = (-2, 7)$, Kendall is trying to decide which point on the line is closest to A. The point $P = (3, 5)$ looks promising. To check that P really is the point on $y = 2x - 1$ that is closest to A, what should Kendall do? Is P closest to A?

120. A triangle has six principal parts — three sides and three angles. The SSS criterion states that three of these items (the sides) determine the other three (the angles). Are there other combinations of three parts that determine the remaining three? In other words, if the class is given three measurements with which to draw and cut out a triangle, which three measurements will guarantee that everyone's triangles will be congruent?

121. Dissect a 1-by-3 rectangle into three pieces that can be reassembled into a square.

122. Let $K = (-2, 1)$ and $M = (3, 4)$. Find coordinates for the two points that divide segment KM into three congruent segments.

123. The components of vector $[24, 7]$ are 24 and 7. Find the components of a vector that is three fifths as long as $[24, 7]$.

124. Let $A = (-5, 2)$ and $B = (19, 9)$. Find coordinates for the point P between A and B that is three fifths of the way from A to B. Find coordinates for the point Q between A and B that is three fifths of the way from B to A.

125. Given the points $K = (-2, 1)$ and $M = (3, 4)$, find coordinates for a point J that makes angle JKM a right angle.

126. When two lines intersect, four angles are formed. It is not hard to believe that the nonadjacent angles in this arrangement are congruent. If you had to prove this to a skeptic, what reasons would you offer?

127. One of the legs of a right triangle is twice as long as the other, and the perimeter of the triangle is 28. Find the lengths of all three sides, to three decimal places.

128. A car traveling east at 45 miles per hour passes a certain intersection at 3 pm. Another car traveling north at 60 miles per hour passes the same intersection 25 minutes later. To the nearest minute, figure out when the cars are exactly 40 miles apart.

129. Find a point on the line $y = 2x - 3$ that is 5 units from the x-axis.

130. Find a point on the line $2x + y = 8$ that is equidistant from the coordinate axes. How many such points are there?

131. A line goes through the points $(2, 5)$ and $(6, -1)$. Let P be the point on this line that is closest to the origin. Calculate the coordinates of P.

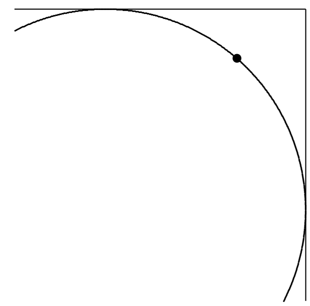
132. The lines defined by $P_t = (4 + 5t, -1 + 2t)$ and $Q_u = (4 - 2u, -1 + 5u)$ intersect perpendicularly. Justify this statement. What are the coordinates of the point of intersection?

133. What number is exactly midway between $23 - \sqrt{17}$ and $23 + \sqrt{17}$? What number is exactly midway between $\dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$ and $\dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$?

134. Given that $P = (-1, -1)$, $Q = (4, 3)$, $A = (1, 2)$, and $B = (7, k)$, find the value of k that makes the line AB **(a)** parallel to line PQ; **(b)** perpendicular to line PQ.

135. Let $A = (-6, -4)$, $B = (1, -1)$, $C = (0, -4)$, and $D = (-7, -7)$. Show that the *opposite* sides of quadrilateral $ABCD$ are parallel. A quadrilateral that has this property is called a *parallelogram*.

136. Ashley saved a distance equal to 80% of the length of the shortest side of a rectangular field by cutting across the diagonal of the field instead of along two of the sides. Find the ratio of the length of the shortest side to the length of the longest.



137. A circular Harkness table is placed in a corner of a room so that it touches both walls. A mark is made on the edge of the table, exactly 18 inches from one wall and 25 inches from the other. What is the radius of the table?

138. If a line intersects the x-axis at $(a, 0)$ and intersects the y-axis at $(0, b)$, at what point does it intersect the line $y = x$?

139. Given $A = (-1, 5)$, $B = (x, 2)$, and $C = (4, -6)$ and the sum of $AB + BC$ is to be a minimum, find the value of x.

140. On the same coordinate-axis system, graph the line defined by $P_t = (3t - 4, 2t - 1)$ and the line defined by $4x + 3y = 18$. The graphs should intersect in the first quadrant.
(a) Calculate P_2, and show that it is not the point of intersection.
(b) Find the value of t for which P_t is on the line $4x + 3y = 18$.

141. Find a vector that translates the line $2x - 3y = 18$ onto the line $2x - 3y = 24$. (There is more than one correct answer.)

142. Let $A = (0, 0)$, $B = (4, 2)$, and $C = (1, 3)$, find the size of angle CAB. Justify your answer.

143. Let $A = (3, 2)$, $B = (1, 5)$, and $P = (x, y)$. Find x- and y-values that make ABP a right angle.

144. (Continuation) Describe the configuration of all such points P.

145. Find coordinates for the vertices of a *lattice rectangle* that is three times as long as it is wide, with none of the sides horizontal.

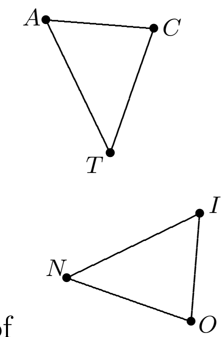
146. The vector that is defined by a directed segment AB is often denoted $\overrightarrow{AB}$. Find components for the following vectors $\overrightarrow{AB}$:
(a) $A = (1, 2)$ and $B = (3, -7)$ **(b)** $A = (2, 3)$ and $B = (2 + 3t, 3 - 4t)$

147. If $A = (-2, 5)$ and $B = (-3, 9)$, find components for the vector that points
(a) from A to B **(b)** from B to A

148. If M is the midpoint of segment AB, how are vectors $\overrightarrow{AM}$, $\overrightarrow{AB}$, $\overrightarrow{MB}$, and $\overrightarrow{BM}$ related?

149. Choose positive integers m and n, with $m < n$. Let $x = 2mn$, $y = n^2 - m^2$, and $z = m^2 + n^2$. It so happens that these three positive integers x, y, and z have a special property. What is the property? Can you prove a general result?

150. Show that the lines $3x - 4y = -8$, $x = 0$, $3x - 4y = 12$, and $x = 4$ form the sides of a rhombus.



151. Suppose that triangle ACT has been shown to be congruent to triangle ION, with vertices A, C, and T corresponding to vertices I , O, and N, respectively. It is customary to record this result by writing $\Delta ACT \cong \Delta ION$. Notice that corresponding vertices occupy corresponding positions in the equation. Let $B = (5, 2)$, $A = (-1, 3)$, $G = (-5, -2)$, $E = (1, -3)$, and $L = (0, 0)$. Using only these five labels, find as many pairs of congruent triangles as you can, and express the congruences accurately.

152. (Continuation) How many ways are there of arranging the six letters of $\Delta ACT \cong \Delta ION$ to express the two-triangle congruence?

153. What can be concluded about triangle ABC if it is given that
(a) $\Delta ABC \cong \Delta ACB$? **(b)** $\Delta ABC \cong \Delta BCA$?

154. Plot points $K = (-4, -3)$, $L = (-3, 4)$, $M = (-6, 3)$, $X = (0, -5)$, $Y = (6, -3)$, and $Z = (5, 0)$. Show that triangle KLM is congruent to triangle XZY. Describe a transformation that transforms KLM onto XZY. Where does this transformation send the point $(-5, 0)$?

155. Positions of three objects are described by the following three pairs of equations

$$\textbf{(a)} \begin{cases} x = 2 - 2t \\ y = 5 + 7t \end{cases} \qquad \textbf{(b)} \begin{cases} x = 4 - 2t \\ y = -2 + 7t \end{cases} \qquad \textbf{(c)} \begin{cases} x = 2 - 2(t+1) \\ y = 5 + 7(t+1) \end{cases}$$

How do the positions of these objects compare at any given moment?

156. Brooks and Avery are running laps around the outdoor track, in the same direction. Brooks completes a lap every 78 seconds while Avery needs 91 seconds for every tour of the track. Brooks (the faster runner) has just passed Avery. How much time will it take for Brooks to overtake Avery again?

157. Alex the geologist is in the desert, 10 km from the nearest point N on a long, straight road. Alex's jeep can do 50 kph on the road, and 30 kph in the desert. Find the shortest time for Alex to reach an oasis that is on the road **(a)** 20 km from N; **(b)** 30 km from N.

158. Robin is moving on the xy-plane according to the rule $(x, y) = (-3 + 8t, 5 + 6t)$, with distance measured in km and time in hours. Casey is following 20 km behind on the same path at the same speed. Write parametric equations describing Casey's position.

159. Is it possible for a line to go through **(a)** no lattice points? **(b)** exactly one lattice point? **(c)** exactly two lattice points? For each answer, either give an example or else explain the impossibility.

160. Describe a transformation that carries the triangle with vertices $A = (0, 0)$, $B = (13, 0)$, and $C = (3, 2)$ onto the triangle with vertices $P = (0, 0)$, $Q = (12, 5)$, and $R = (2, 3)$. Where does your transformation send the point $(6, 0)$?

161. Given $A = (6, 1)$, $B = (1, 3)$, and $C = (4, 3)$, find a lattice point P that makes $\overrightarrow{CP}$ perpendicular to $\overrightarrow{AB}$.

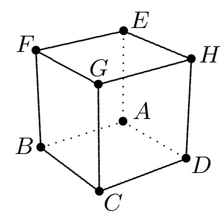
162. (Continuation) Describe the set of points P for which $\overrightarrow{AB}$ and $\overrightarrow{CP}$ are perpendicular.

163. Let $A = (0, 0)$, $B = (1, 2)$, $C = (6, 2)$, $D = (2, -1)$, and $E = (1, -3)$. Show that angle CAB is the same size as angle EAD.

164. Let $A = (-2, 4)$ and $B = (7, -2)$. Find the point Q on the line $y = 2$ that makes the total distance $AQ + BQ$ as small as possible.

165. (Continuation) Let $A = (-2, 4)$ and $B = (7, 6)$. Find the point P on the line $y = 2$ that makes the total distance $AP + BP$ as small as possible.

166. An ant is sitting at F, one of the eight vertices of a solid cube. It needs to crawl to vertex D as fast as possible. Find one of the shortest routes. How many are there?

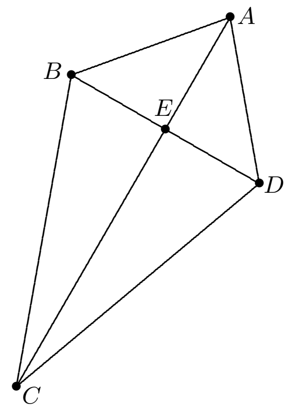


167. A particle moves according to $(x, y) = (6 - t, -1 + 3t)$. For what value of t is the particle closest to the point $(-2, 0)$?

168. Two automobiles each travel 60 km at steady rates. One car goes 6 kph faster than the other, thereby taking 20 minutes less time for the trip. Find the rate of the slower car.

169. What do the descriptions of position defined by equations $P_t = (-2 + t, 3 + 2t)$ and $Q_u = (4 + 3u, -1 + 6u)$ have in common? How do they differ?

170. Here are some examples of proofs that do not use coordinates. They all show how specific given information can be used to logically deduce new information. Each example concerns a kite $ABCD$, for which $AB = AD$ and $BC = DC$ is the given information. The first two proofs show that diagonal AC creates angles BAC and DAC of the same size. The first proof consists of simple text; the second proof is written symbolically as an outline; this statement-reason form is sometimes called a "two-column proof."



Proof A: Because $AB = AD$ and $BC = DC$, and because the segment AC is shared by triangles ABC and ADC, it follows from the SSS criterion that these triangles are congruent. Thus, the corresponding parts of these congruent triangles are also congruent (often abbreviated to CPCTC, as in proof B below). In particular, angles BAC and DAC are the same size.

QED

Proof B:

$AB = AD$	given
$BC = DC$	given
$AC = AC$	shared side
$\Delta ABC \cong \Delta ADC$	SSS
$\angle BAC = \angle DAC$	CPCTC

□

In the fourth line, why would writing $\Delta ABC \cong \Delta ACD$ have been incorrect?

171. (Continuation) Prove that angles ABC and ADC are the same size.

172. (Continuation) Now let E be the intersection of diagonals AC and BD. The diagram makes it look like the diagonals intersect perpendicularly. Here are two proofs of this conjecture, each building on the result just proved.

Proof C: It is known that angles BAC and DAC are the same size (proof A). Because $AB = AD$ is given, and because edge AE is common to triangles BEA and DEA, it follows from the SAS criterion that these triangles are congruent. Their corresponding angles BEA and DEA must therefore be the same size. They are also supplementary, which makes them right angles, by definition.

□

Proof D:

$AB = AD$	given
$\angle BAE = \angle DAE$	proof B
$AE = AE$	shared side
$\Delta ABE \cong \Delta ADE$	SAS
$\angle BEA = \angle DEA$	CPCTC
$\angle BEA$ and $\angle DEA$ supplementary	E is on BD
$\angle BEA$ is right	definition of right angle

QED

Prove that AC bisects BD.

173. An *altitude* of a triangle is a segment that joins one of the three vertices to a point on the line that contains the opposite side, the intersection being *perpendicular*. For example, consider the triangle whose vertices are $A = (0, 0)$, $B = (8, 0)$, and $C = (4, 12)$.
(a) Find the length of the altitude from C to side AB. What is the area of ABC?
(b) Find an equation for the line that contains the altitude from A to side BC.
(c) Find an equation for the line BC.
(d) Find coordinates for the point F where the altitude from A meets side BC. It is customary to call F the *foot* of the altitude from A.
(e) Find the length of the altitude from A to side BC.
(f) As a check on your work, calculate BC and multiply it by your answer to part (e). You should be able to predict the result.
(g) It is possible to deduce the length of the altitude from B to side AC from what you have already calculated. Show how.

174. Let $A = (0, 0)$, $B = (8, 1)$, $C = (5, -5)$, $P = (0, 3)$, $Q = (7, 7)$, and $R = (1, 10)$. Prove that angles ABC and PQR have the same size.

175. (Continuation) Let D be the point on segment AB that is exactly 3 units from B, and let T be the point on segment PQ that is exactly 3 units from Q. What evidence can you give for the congruence of triangles BCD and QRT?

176. Find a point on the line $x + 2y = 8$ that is equidistant from the points $(3, 8)$ and $(9, 6)$.

177. Graph the line that is described parametrically by $(x, y) = (2t, 5 - t)$, then
(a) confirm that the point corresponding to $t = 0$ is exactly 5 units from $(3, 9)$;
(b) write a formula in terms of t for the distance from $(3, 9)$ to $(2t, 5 - t)$;
(c) find the other point on the line that is 5 units from $(3, 9)$;
(d) find the point on the line that minimizes the distance to $(3, 9)$.

178. How large a square can be put inside a right triangle whose legs are 5 cm and 12 cm?

179. You are one mile from the railroad station, and your train is due to leave in ten minutes. You have been walking at a steady rate of 3 mph, and you can run at 8 mph if you have to. For how many more minutes can you continue walking, until it becomes necessary for you to run the rest of the way to the station?

180. If a quadrilateral is equilateral, its diagonals are perpendicular. True or false? Why?

181. The diagonals AC and BD of quadrilateral $ABCD$ intersect at O. Given the information $AO = BO$ and $CO = DO$, what can you deduce about the lengths of the sides of the quadrilateral? Prove your response.

182. Let $A = (7, 7)$, $B = (5, 1)$, and $P_t = (6 + 3t, 4 - t)$. Plot A and B. Choose two values for t and plot the resulting points P_t, which should look equidistant from A and B. Make calculations to confirm the equidistance.

183. Let $A = (-2, 3)$, $B = (6, 7)$, and $C = (-1, 6)$.
(a) Find an equation for the perpendicular bisector of AB.
(b) Find an equation for the perpendicular bisector of BC.
(c) Find coordinates for a point K that is equidistant from A, B, and C.

184. A segment from one of the vertices of a triangle to the midpoint of the opposite side is called a *median*. Consider the triangle defined by $A = (-2, 0)$, $B = (6, 0)$, and $C = (4, 6)$.
(a) Find an equation for the line that contains the median drawn from A to BC.
(b) Find an equation for the line that contains the median drawn from B to AC.
(c) Find coordinates for G, the intersection of the medians from A and B.
(d) Let M be the midpoint of AB. Determine whether or not M, G, and C are collinear.

185. The transformation defined by $\mathcal{T}(x, y) = (y + 2, x - 2)$ is a reflection. Verify this by calculating the effect of $\mathcal{T}$ on the triangle formed by $P = (1, 3)$, $Q = (2, 5)$, and $R = (6, 5)$. Sketch triangle PQR, find coordinates for the *image points* P', Q', and R', and sketch the *image triangle* $P'Q'R'$. Then identify the mirror line and add it to your sketch. Notice that triangle PQR is labeled in a clockwise sense; what about the labels on triangle $P'Q'R'$?

186. In quadrilateral $ABCD$, it is given that $AB = CD$ and $BC = DA$. Prove that angles ACD and CAB are the same size. N.B. If a polygon has more than three vertices, the *labeling convention* is to place the letters around the polygon in the order that they are listed. Thus AC should be one of the diagonals of $ABCD$.

187. Maintaining constant speed and direction for an hour, Whitney traveled from $(-2, 3)$ to $(10, 8)$. Where was Whitney after 35 minutes? What distance did Whitney cover in those 35 minutes?

188. A *direction vector* for a line is any vector that joins two points on that line. Find a direction vector for $2x + 5y = 8$. It is not certain that you and your classmates will get exactly the same answer. How should your answers be related, however?

189. (Continuation) Show that $[b, -a]$ is a direction vector for the line $ax + by = c$.

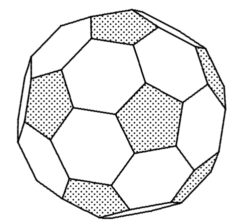
190. (Continuation) Show that any direction vector for the line $ax + by = c$ must be perpendicular to $[a, b]$. We refer to the vector $[a, b]$ as a *normal vector* to the line.

191. A particle moves according to the equation $(x, y) = (1, 2) + t[4, 3]$. Let P be the point where the path of this particle intersects the line $4x + 3y = 16$. Find coordinates for P, then explain why P is the point on $4x + 3y = 16$ that is closest to $(1, 2)$.

192. True or false? $\sqrt{4x} + \sqrt{9x} = \sqrt{13x}$

193. The line $3x+2y = 16$ is the perpendicular bisector of the segment AB. Find coordinates of point B, given that **(a)** $A = (-1, 3)$; **(b)** $A = (0, 3)$.

194. (Continuation) Point B is called the *reflection of* A *across the line* $3x + 2y = 16$; sometimes B is simply called the *image* of A, often denoted A'. Explain this terminology. Using the same line, find the image C' for the points **(a)** $C = (10, 6)$ and **(b)** $C = (7, 2)$. How can you check your answer?



195. A cube has 8 vertices, 12 edges, and 6 square faces. A soccer ball (also known as a *buckyball* or *truncated icosahedron*) has 12 pentagonal faces and 20 hexagonal faces. How many vertices and how many edges does a soccer ball have?

196. A rhombus has 25-cm sides, and one diagonal is 14 cm long. How long is the other diagonal?

197. Let $A = (0, 0)$ and $B = (12, 5)$, and let C be the point on segment AB that is 8 units from A. Find coordinates for C.

198. Find the point on the y-axis that is equidistant from $A = (0, 0)$ and $B = (12, 5)$.

199. Let $A = (1, 4)$, $B = (8, 0)$, and $C = (7, 8)$. Find the area of triangle ABC.

200. Sketch triangle PQR, where $P = (1, 1)$, $Q = (1, 2)$, and $R = (3, 1)$. For each of the following, apply the given transformation $\mathcal{T}$ to the vertices of triangle PQR, sketch the image triangle $P'Q'R'$, then decide which of the terms *reflection*, *rotation*, *translation*, or *glide-reflection* accurately describes the action of $\mathcal{T}$. Provide appropriate detail to justify your choices.

(a) $\mathcal{T}(x, y) = (x + 3, y - 2)$ **(b)** $\mathcal{T}(x, y) = (y, x)$
(c) $\mathcal{T}(x, y) = (-x + 2, -y + 4)$ **(d)** $\mathcal{T}(x, y) = (x + 3, -y)$

201. Prove that one of the diagonals of a kite bisects two of the angles of the kite. What about the other diagonal — must it also be an *angle bisector*? Explain your response.

202. Let $A = (2, 9)$, $B = (6, 2)$, and $C = (10, 10)$. Verify that segments AB and AC have the same length. Measure angles ABC and ACB. On the basis of your work, propose a general statement that applies to any triangle that has two sides of equal length. Prove your assertion, which might be called the *Isosceles-Triangle Theorem*.

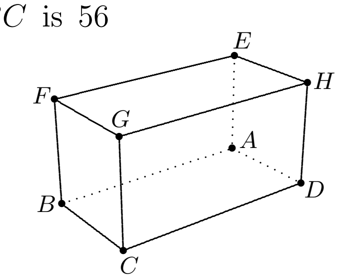
203. If the diagonals of a quadrilateral bisect each other, then any two nonadjacent sides of the figure must have the same length. Prove that this is so.

204. A geometric transformation is called an *isometry* if it preserves distances, in the following sense: The distance from M to N must be the same as the distance from M' to N', for any two points M and N and their respective images M' and N'. You have already shown in a previous exercise that any translation is an isometry. Now let $M = (a, b)$, $N = (c, d)$, $M' = (a, -b)$, and $N' = (c, -d)$. Confirm that segments MN and $M'N'$ have the same length, thereby showing that a certain transformation $\mathcal{T}$ is an isometry. What type of transformation is $\mathcal{T}$?

205. Use the distance formula to show that $\mathcal{T}(x, y) = (-y, x)$ is an isometry.

206. Triangle ABC is isosceles, with $AB = BC$, and angle BAC is 56 degrees. Find the remaining two angles of this triangle.

207. Triangle ABC is isosceles, with $AB = BC$, and angle ABC is 56 degrees. Find the remaining two angles of this triangle.



208. An ant is sitting at F, one of the vertices of a solid rectangular block. Edges AD and AE are each half the length of edge AB. The ant needs to crawl to vertex D as fast as possible. Find one of the shortest routes. How many are there?

209. Suppose that vectors $[a, b]$ and $[c, d]$ are perpendicular. Show that $ac + bd = 0$.

210. Suppose that $ac + bd = 0$. Show that vectors $[a, b]$ and $[c, d]$ are perpendicular. The number $ac + bd$ is called the *dot product* of the vectors $[a, b]$ and $[c, d]$.

211. Let $A = (0, 0)$, $B = (4, 3)$, $C = (2, 4)$, $P = (0, 4)$, and $Q = (-2, 4)$. Decide whether angles BAC and PAQ are the same size (congruent, that is), and give your reasons.

212. Let $A = (-4, 0)$, $B = (0, 6)$, and $C = (6, 0)$.
(a) Find equations for the three lines that contain the altitudes of triangle ABC.
(b) Show that the three altitudes are *concurrent*, by finding coordinates for their common point. The point of concurrence is called the *orthocenter* of triangle ABC.

213. The equation $y - 5 = m(x - 2)$ represents a line, no matter what value m has.
(a) What do all these lines have in common?
(b) When $m = -2$, what are the x- and y-intercepts of the line?
(c) When $m = -1/3$, what are the x- and y-intercepts of the line?
(d) When $m = 2$, what are the x- and y-intercepts of the line?
(e) For what values of m are the axis intercepts both positive?

214. Find the area of the triangle whose vertices are $A = (-2, 3)$, $B = (6, 7)$, and $C = (0, 6)$.

215. If triangle ABC is isosceles, with $AB = AC$, then the medians drawn from vertices B and C must have the same length. Write a *two-column proof* of this result.

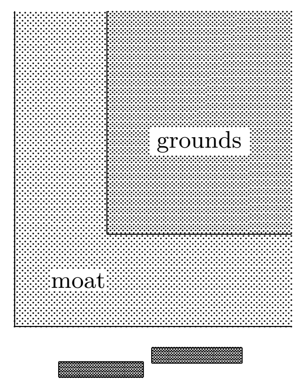
216. Let $A = (-4, 0)$, $B = (0, 6)$, and $C = (6, 0)$.
(a) Find equations for the three medians of triangle ABC.
(b) Show that the three medians are concurrent, by finding coordinates for their common point. The point of concurrence is called the *centroid* of triangle ABC.

217. Given points $A = (0, 0)$ and $B = (-2, 7)$, find coordinates for points C and D so that $ABCD$ is a square.

218. Given the transformation $\mathcal{F}(x, y) = (-0.6x - 0.8y, 0.8x - 0.6y)$, Shane calculated the image of the isosceles right triangle formed by $S = (0, 0)$, $H = (0, -5)$, and $A = (5, 0)$, and declared that $\mathcal{F}$ is a reflection. Morgan instead calculated the image of the *scalene* (non-isosceles) triangle formed by $M = (7, 4)$, $O = (0, 0)$, and $R = (7, 1)$, and concluded that $\mathcal{F}$ is a rotation. Who was correct? Explain your choice, and account for the disagreement.

219. Let $A = (0, 12)$ and $B = (25, 12)$. If possible, find coordinates for a point P on the x-axis that makes angle APB a right angle.

220. Brett and Jordan are out driving in the coordinate plane, each on a separate straight road. The equations $B_t = (-3, 4) + t[1, 2]$ and $J_t = (5, 2) + t[-1, 1]$ describe their respective travels, where t is the number of minutes after noon.
(a) Make a sketch of the two roads, with arrows to indicate direction of travel.
(b) Where do the two roads intersect?
(c) How fast is Brett going? How fast is Jordan going?
(d) Do they collide? If not, who gets to the intersection first?



221. A castle's grounds are surrounded by a rectangular moat, which is a uniform width of 12 feet. A corner is shown in the top view at right. The problem is to get across the moat to dry land on the other side, without using the drawbridge. To work with, you have only two rectangular planks, one with length 11 feet and another with length 11 feet, 9 inches. Show how to use the planks to get across.

222. Find k so that the vectors $[4, -3]$ and $[k, -6]$
(a) point in the same direction; **(b)** are perpendicular.

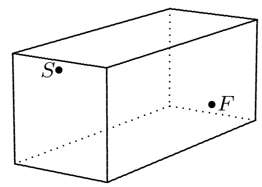
223. The lines $3x + 4y = 12$ and $3x + 4y = 72$ are parallel. Explain why. Find the distance that separates these lines. The distance between two lines is defined to be the shortest possible distance between the lines.

224. Give an example of an equiangular polygon that is not equilateral.

225. Find coordinates for a point on the line $4y = 3x$ that is 8 units from $(0, 0)$.

226. An object moves with constant *velocity* (which means constant speed and direction) from $(-3,1)$ to $(5,7)$, taking five seconds for the trip.
(a) What is the speed of the object?
(b) Where does the object cross the y-axis?
(c) Where is the object three seconds after it leaves $(-3,1)$?

227. A spider lived in a room that measured 30 feet long by 12 feet wide by 12 feet high. One day, the spider spied an incapacitated fly across the room, and of course wanted to crawl to it as quickly as possible. The spider was on an end wall, one foot from the ceiling and six feet from each of the long walls. The fly was stuck one foot from the floor on the opposite wall, also midway between the two long walls. Knowing some geometry, the spider cleverly took the shortest possible route to the fly and ate it for lunch. How far did the spider crawl?



228. Told to investigate the transformation $\mathcal{T}(x,y)=(x+3,2y+1)$, Morgan calculated the images of $P=(1,5)$ and $Q=(-3,5)$. Because PQ and $P'Q'$ are equal, Morgan declared that $\mathcal{T}$ is an isometry. Shane disagreed with this conclusion. Who is correct, and why?

229. Find the area of the parallelogram whose vertices are $(0,0)$, $(7,2)$, $(8,5)$, and $(1,3)$.

230. Let $A=(0,0)$ and $B=(12,5)$. There are points on the y-axis that are twice as far from B as they are from A. Make a diagram that shows these points, and use it to estimate their coordinates. Then use algebra to find them exactly.

231. Given the points $A=(0,0)$, $B=(7,1)$, and $D=(3,4)$, find coordinates for the point C that makes quadrilateral $ABCD$ a parallelogram. What if the question had requested $ABDC$ instead?

232. Find a vector that is perpendicular to the line $3x-4y=6$.

233. Measurements are made on quadrilaterals $ABCD$ and $PQRS$, and it is found that angles A, B, and C are the same size as angles P, Q, and R, respectively, and that sides AB and BC are the same length as PQ and QR, respectively. Is this enough evidence to conclude that the quadrilaterals $ABCD$ and $PQRS$ are congruent? Explain.

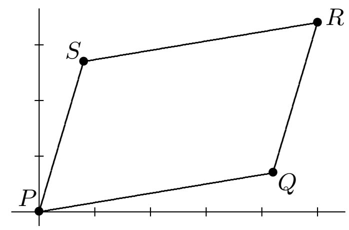
234. Let $P=(-1,3)$. Find the point Q for which the line $2x+y=5$ serves as the perpendicular bisector of segment PQ.

235. Let $A=(3,4)$, $B=(0,-5)$, and $C=(4,-3)$. Find equations for the perpendicular bisectors of segments AB and BC, and coordinates for their common point K. Calculate lengths KA, KB, and KC. Why is K also on the perpendicular bisector of segment CA?

236. (Continuation) A circle centered at K can be drawn so that it goes through all three vertices of triangle ABC. Explain. This is why K is called the *circumcenter* of the triangle. In general, how do you locate the circumcenter of a triangle?

237. The equation $y - 5 = m(x - 2)$ represents a line, no matter what value m has.
(a) What are the x- and y-intercepts of this line?
(b) For what value of m does this line form a triangle of area 36 with the positive axes?
(c) Show that the area of a first-quadrant triangle formed by this line must be at least 20.

238. The figure at right shows a parallelogram $PQRS$, three of whose vertices are $P = (0, 0)$, $Q = (a, b)$, and $S = (c, d)$.
(a) Find the coordinates of R.
(b) Find the area of $PQRS$, and simplify your formula.



239. Working against a 1-km-per-hour current, some members of the Outing Club paddled 7 km up the Exeter River one Saturday last spring and made camp. The next day, they returned downstream to their starting point, aided by the same one-km-per-hour current. They paddled for a total of 6 hours and 40 minutes during the round trip. Use this information to figure out how much time the group would have needed to make the trip if there had been no current.

240. Decide whether the transformation $\mathcal{T}(x, y) = \left(2x, \frac{1}{2}y\right)$ is an isometry, and give your reasons.

241. Find points on the line $3x + 5y = 15$ that are equidistant from the coordinate axes.

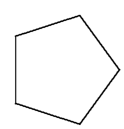
242. Plot all points that are 3 units from the x-axis. Describe the configuration.

243. Plot all points that are 3 units from the x-axis and 3 units from $(5, 4)$. How many did you find?

244. In triangle ABC, it is given that $CA = CB$. Points P and Q are marked on segments CA and CB, respectively, so that angles CBP and CAQ are the same size. Prove that $CP = CQ$.

245. (Continuation) Segments BP and AQ intersect at K. Explain why you can be sure that quadrilateral $CPKQ$ is a kite. You might want to consider triangles AKP and BKQ.

246. A polygon that is both equilateral and equiangular is called *regular*. Prove that all diagonals of a regular *pentagon* (five sides) have the same length.



247. Find coordinates for the point equidistant from $(-1, 5)$, $(8, 2)$, and $(6, -2)$.

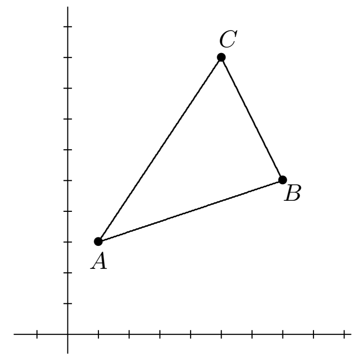
248. Find coordinates for the point where line $(x, y) = (3+2t, -1+3t)$ meets line $y = 2x-5$.

249. Find an equation for the line that goes through $(5, 2)$ and that forms a triangle in the first quadrant that is just large enough to enclose the 4-by-4 square in the first quadrant that has two of its sides on the coordinate axes.

250. Find the area of the parallelogram whose vertices are $(2, 5)$, $(7, 6)$, $(10, 10)$, and $(5, 9)$.

251. Let $E = (2, 7)$ and $F = (10, 1)$. On the line through E and F, there are two points that are 3 units from E. Find coordinates for both of them.

252. Let $A = (1, 3)$, $B = (7, 5)$, and $C = (5, 9)$. Answer the item below that is determined by *the first letter of your first name*. Find coordinates for the requested point.



(a-e) Show that the three medians of triangle ABC are concurrent at a point G.
(f-m) Show that the three altitudes of triangle ABC are concurrent at a point H.
(n-z) Show that the perpendicular bisectors of the sides of triangle ABC are concurrent at a point K. What special property does K have?

253. (Continuation for class discussion) It looks like G, H, and K are collinear. Are they?

254. Show that the following transformations are isometries, and identify their type:
(a) $\mathcal{T}(x, y) = (-x, y + 2)$ **(b)** $\mathcal{T}(x, y) = (0.6x - 0.8y, 0.8x + 0.6y)$

255. Find coordinates for a point that is three times as far from the origin as $(2, 3)$ is. Describe the configuration of all such points.

256. What are the axis intercepts of the line described by $P_t = (5 + 3t, -2 + 4t)$?

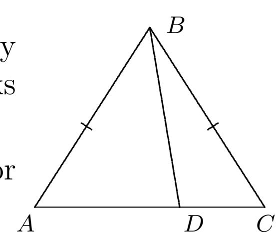
257. Simplify equation $\sqrt{(x-3)^2 + (y-5)^2} = \sqrt{(x-7)^2 + (y+1)^2}$. Interpret your result.

258. How large an equilateral triangle can you fit inside a 2-by-2 square?

259. Plot the points $K = (0, 0)$, $L = (7, -1)$, $M = (9, 3)$, $P = (6, 7)$, $Q = (10, 5)$, and $R = (1, 2)$. You will see that the triangles KLM and RPQ are congruent. Find coordinates for the point in triangle KLM that corresponds to $(3, 4)$ in triangle RPQ.

260. Find a fourth-quadrant point that is equally distant from $(4, 1)$ and the y-axis.

261. Use the diagram to help you explain why SSA evidence is not by itself sufficient to justify the congruence of triangles. The tick marks designate segments that have the same length.



262. The diagonals of a kite are 6 cm and 12 cm long. Is it possible for the lengths of the sides of this kite to be in a 2-to-1 ratio?

263. Translate the line $5x + 7y = 35$ by vector $[3, 10]$. Find an equation for the new line.

264. Let $A = \left(\sqrt{15}, \sqrt{11}\right)$, $B = \left(\sqrt{11}, -\sqrt{15}\right)$, $C = (1, 2)$, and $D = (7, 6)$. Is there an isometry that transforms segment AB onto segment CD? Explain.

265. You have recently seen that there is no completely reliable SSA criterion for congruence. If the angle part of such a correspondence is a *right* angle, however, the criterion *is* reliable. Justify this so-called *hypotenuse-leg* criterion (which is abbreviated HL).

266. It is given that $a + b = 6$ and $ab = 7$.
(a) Find the value of $a^2 + b^2$. Can you do this without finding values for a and b?
(b) Make up a geometry word problem that corresponds to the question in part (a).

267. Avery can run at 10 uph. The bank of a river is represented by the line $4x + 3y = 12$, and Avery is at $(7, 5)$. How much time does Avery need to reach the river?

268. Find an equation for the line through point $(7, 9)$ that is perpendicular to vector $[5, -2]$.

269. Describe a transformation that carries the triangle with vertices $R = (1, 2)$, $P = (6, 7)$, and $Q = (10, 5)$ onto the triangle with vertices $K = (0, 0)$, $L = (7, -1)$, and $M = (9, 3)$. Where does your transformation send **(a)** $(4, 5)$? **(b)** $(7, 5)$?

270. Apply the transformation $\mathcal{T}(x, y) = \left(\frac{\sqrt{3}}{2}x + \frac{1}{2}y\,, -\frac{1}{2}x + \frac{\sqrt{3}}{2}y\right)$ to the triangle whose vertices are $(0, 0)$, $(4, 0)$, and $(0, 8)$. Is $\mathcal{T}$ an isometry?

271. A triangle that has a 13-inch side, a 14-inch side, and a 15-inch side has an area of 84 square inches. Accepting this fact, find the lengths of all three altitudes of this triangle.

272. Find the point of intersection of the lines $P_t = (-1+3t, 3+2t)$ and $Q_r = (4-r, 1+2r)$.

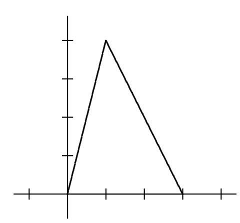
273. Find the area of the triangle having sides 10, 10, and 5.

274. Apply the transformation $\mathcal{T}(x, y) = (2x+3y, -x+y)$ to the unit square, whose vertices are $(0, 0)$, $(1, 0)$, $(0, 1)$, and $(1, 1)$. Even though $\mathcal{T}$ is not a reflection, it is customary to call the resulting figure the *image* of the square. What kind of figure is it?

275. Explain why an isometry always transforms a right triangle onto a right triangle.

276. (Continuation) Consider a transformation $\mathcal{T}$ for which the image of the x-axis is the line $2x + 3y = 6$ and the image of the y-axis is the line $x + 7y = 7$. What must be the image of the origin? Could $\mathcal{T}$ be an isometry?

277. (Continuation) Consider a transformation $\mathcal{T}$ for which the image of the x-axis is the line $3x - 2y = 12$ and the image of the y-axis is the line $2x + 3y = 6$. What must be the image of the origin? Could $\mathcal{T}$ be an isometry?



278. Find the lengths of *all* the altitudes of the triangle whose vertices are $(0, 0)$, $(3, 0)$, and $(1, 4)$.

279. Form a scalene triangle using three lattice points of your choosing. Verify that the medians of your triangle are concurrent.

280. Let $P = (2, 7)$, $B = (6, 11)$, and $M = (5, 2)$. Find a point D that makes $\overrightarrow{PB} = \overrightarrow{DM}$. What can you say about quadrilateral $PBMD$?

281. When translation by vector $[2, 5]$ is followed by translation by vector $[5, 7]$, the net result can be achieved by applying a *single* translation; what is its vector?

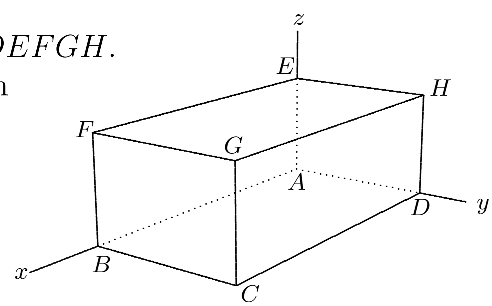
282. Given that $(-1, 4)$ is the reflected image of $(5, 2)$, find an equation for the line of reflection.

283. Draw a parallelogram whose adjacent edges are determined by vectors $[2, 5]$ and $[7, -1]$, placed so that they have a common initial point. This is called placing vectors *tail-to-tail*. Find the area of the parallelogram.

284. Point $(0, 1)$ is reflected across the line $2x + 3y = 6$. Find coordinates for its image.

285. A stop sign — a regular *octagon* — can be formed from a 12-inch square sheet of metal by making four straight cuts that snip off the corners. How long, to the nearest 0.01 inch, are the sides of the resulting polygon?

286. The diagram shows a rectangular box named $ABCDEFGH$. Notice that $A = (0, 0, 0)$, and that B, D, and E are on the coordinate axes. Given that $G = (6, 3, 2)$, find
(a) coordinates for the other six vertices;
(b) the lengths AH, AC, AF, and AG.



287. The edges of a rectangular solid are parallel to the coordinate axes, and it has the points $(0, 0, 0)$ and $(8, 4, 4)$ as diagonally opposite vertices. Make a sketch, labeling each vertex with its coordinates, then find **(a)** the distance from $(8, 4, 4)$ to $(0, 0, 0)$ and **(b)** the distance from $(8, 4, 4)$ to the z-axis.

288. Find components for the vector that points from $(1, 1, 1)$ to $(2, 3, 4)$. Then find the distance from $(1, 1, 1)$ to $(2, 3, 4)$ by finding the length of this vector.

289. There are many points in the first quadrant (of the xy-plane) that are the same distance from the x-axis as they are from the point $(0, 2)$. Make a sketch that shows several of them. Use your ruler to check that the points you plotted satisfy the required equidistance property.

290. Find the distance from the origin to **(a)** $(3, -2, 2)$; **(b)** (a, b, c).

291. Find the area of a triangle formed by placing the vectors $[3, 6]$ and $[7, 1]$ tail-to-tail.

292. (Continuation) Describe your triangle using a different pair of vectors.

293. (Continuation) Find the length of the longest altitude of your triangle.

294. The diagonals of quadrilateral $ABCD$ intersect perpendicularly at O. What can be said about quadrilateral $ABCD$?

295. The *sum* of two vectors $[a, b]$ and $[p, q]$ is defined as $[a+p, b+q]$. Find the components of the vectors **(a)** $[2, 3] + [-7, 5]$; **(b)** $[3, 4, 8] + [4, 2, 5]$.

296. Find x so that the distance from $(x, 3, 6)$ to the origin is 9 units.

297. Write an equation that says the point (x, y, z) is 3 units from the origin. Describe the configuration of all such points.

298. Choose coordinates for three non-collinear points A, B, and C. Calculate components for the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$, and $\overrightarrow{AB} + \overrightarrow{AC}$, and then translate point A by vector $\overrightarrow{AB} + \overrightarrow{AC}$. Call the new point D. What kind of quadrilateral is $ABDC$?

299. What do you call **(a)** an *equiangular quadrilateral*? **(b)** an *equilateral quadrilateral*?

300. Let $A = (5, -3, 6)$, $B = (0, 0, 0)$, and $C = (3, 7, 1)$. Show that ABC is a right angle.

301. Find components for a vector of length 21 that points in the same direction as $[2, 3, 6]$.

302. Find coordinates for the point on segment KL that is 5 units from K, where
(a) $K = (0, 0, 0)$ and $L = (4, 7, 4)$; **(b)** $K = (3, 2, 1)$ and $L = (7, 9, 5)$.

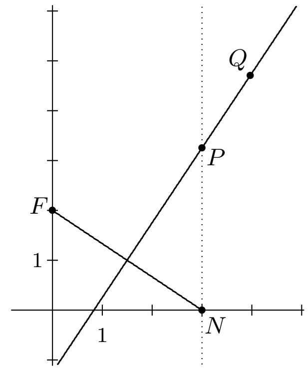
303. Simplify the sum of vectors $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$.

304. In quadrilateral $ABCD$, it is given that $\overrightarrow{AB} = \overrightarrow{DC}$. What kind of a quadrilateral is $ABCD$? What can be said about the vectors $\overrightarrow{AD}$ and $\overrightarrow{BC}$?

305. Let $\mathbf{u} = [5, 2]$ and $\mathbf{v} = [1, -3]$. On a single set of axes, starting at a lattice point of your choice, draw the vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{u}+\mathbf{v}$, $\mathbf{u}-\mathbf{v}$, $\mathbf{u}+2\mathbf{v}$, and $2\mathbf{u}-3\mathbf{v}$. All vectors should originate at your chosen lattice point.

306. Draw a parallelogram. Choose one of its vertices and let $\mathbf{u}$ and $\mathbf{v}$ be the vectors defined by the sides that originate at that vertex. Draw $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$. The vectors $\mathbf{u}$ and $\mathbf{v}$ represent the sides of the parallelogram; what do $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$ represent?

307. Let $F = (0, 2)$ and $N = (3, 0)$. Find coordinates for the point P where the perpendicular bisector of segment FN intersects the line $x = 3$.



308. (Continuation) Choose a new $N \neq (3, 0)$ on the x-axis, and repeat the calculation of P: Draw the line through N that is parallel to the y-axis, and find the intersection of this line with the perpendicular bisector of FN. Explain why there is always an intersection point P, no matter what N is chosen.

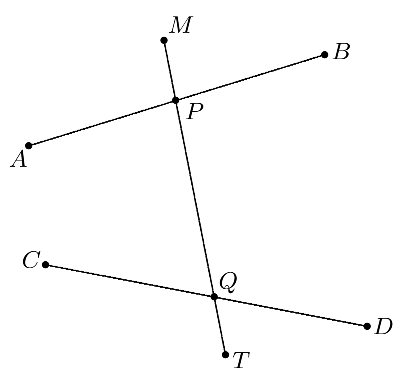
309. (Continuation) Given $Q \neq P$ on the perpendicular bisector of FN, show that the distance from Q to F exceeds the distance from Q to the x-axis. Why was it necessary to exclude the case $Q = P$?

310. The edges of a rectangular solid are parallel to the coordinate axes, and it has the points $(2, 4, 4)$ and $(6, 9, 1)$ as diagonally opposite vertices. Make a sketch, labeling each vertex with its coordinates, then find **(a)** the dimensions of the solid and **(b)** the length of its diagonal.

311. Show that the vectors $[5, -3, 6]$ and $[3, 7, 1]$ are perpendicular.

312. Asked to reflect the point $P = (4, 0)$ across the mirror line $y = 2x$, Aubrey reasoned this way: First mark the point $A = (1, 2)$ on the line, then use the vector $[-3, 2]$ from P to A to reach from A to $P' = (-2, 4)$, which is the requested image. Does this make sense to you? Explain.

313. The diagram at right shows lines APB and CQD intersected by line $MPQT$, which is called a *transversal*. There are two groups of angles: one group of four angles with vertex at P, and another group with vertex at Q. There is special terminology to describe pairs of angles, *one from each group*. If the angles are on different sides of the transversal, they are called *alternate*, for example APM and PQD. Angle BPQ is an *interior* angle because it is between the lines AB and CD, and angle CQT is *exterior*. Thus angles APQ and PQD are called *alternate interior*, while angles MPB and CQT are called *alternate exterior*. On the other hand, the pair of angles MPB and PQD — which are non-alternate angles, one interior, and the other exterior — is called *corresponding*. Refer to the diagram and name



(a) the other pair of alternate interior angles;
(b) the other pair of alternate exterior angles;
(c) the angles that correspond to CQT and to TQD.

314. Write an equation that says that points $(0, 0, 0)$, (a, b, c), and (m, n, p) form a right triangle, the right angle being *at the origin*. Simplify your equation as much as you can.

315. Mark points $A = (1, 7)$ and $B = (6, 4)$ on your graph paper. Use your protractor to draw two lines of positive slope that make 40-degree angles with line AB — one through A and one through B. What can you say about these two lines, and how can you be sure?

316. If one pair of alternate interior angles is equal, what can you say about the two lines that are crossed by the transversal? If one pair of corresponding angles is equal, what can you say about the two lines that are crossed by the transversal?

317. If it is known that one pair of alternate interior angles is equal, what can be said about **(a)** the other pair of alternate interior angles? **(b)** either pair of alternate exterior angles? **(c)** any pair of corresponding angles? **(d)** either pair of non-alternate interior angles?

318. You probably know that the sum of the angles of a triangle is a straight angle. One way to confirm this is to draw a line through one of the vertices, parallel to the opposite side. This creates some alternate interior angles. Finish the demonstration.

319. Suppose that two of the angles of triangle ABC are known to be congruent to two of the angles of triangle PQR. What can be said about the third angles?

320. Suppose that $ABCD$ is a square, and that CDP is an equilateral triangle, with P *outside* the square. What is the size of angle PAD?

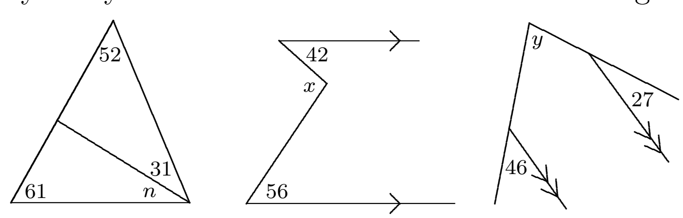
321. Write an equation that says that vectors $[a, b, c]$ and $[m, n, p]$ are perpendicular.

322. Triangle ABC is isosceles, with AB congruent to AC. *Extend* segment BA to a point T (in other words, A should be between B and T). Prove that angle TAC must be twice the size of angle ABC. Angle TAC is called one of the *exterior angles* of triangle ABC.

323. If ABC is any triangle, and TAC is one of its exterior angles, then what can be said about the size of angle TAC, in relation to the other angles of the figure?

324. Given triangle ABC, with $AB = AC$, extend segment AB to a point P so that $BP = BC$. In the resulting triangle APC, show that angle ACP is exactly three times the size of angle APC. (By the way, notice that extending segment AB does *not* mean the same thing as extending segment BA.)

325. Given an arbitrary triangle, what can you say about the *sum* of the three exterior angles, one for each vertex of the triangle?



326. In the diagrams shown at right, the goal is to find the sizes of the angles marked with letters, using the given numerical information. Angles are measured in degrees. Notice the custom of marking arrows on lines to indicate that they are known to be parallel.

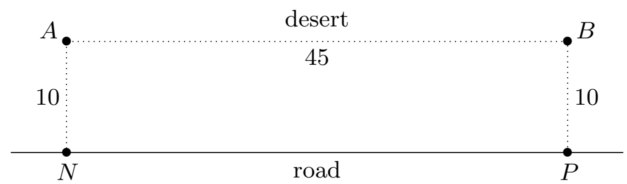
327. Triangle ABC has a 34-degree angle at A. The bisectors of angles B and C meet at point I. What is the size of angle BIC? Answer this question **(a)** assuming that ABC is right; **(b)** assuming that ABC is isosceles; **(c)** choosing sizes for angles B and C. Hmm ...

328. Give an example of a nonzero vector that is perpendicular to $[5, 7, 4]$.

329. Recall that a quadrilateral that has two pairs of parallel opposite sides is called a *parallelogram*. What can be said about the angles of such a figure?

330. Let $ABCD$ be a parallelogram. **(a)** Express $\overrightarrow{AC}$ in terms of $\overrightarrow{AB}$ and $\overrightarrow{BC}$. **(b)** Express $\overrightarrow{AC}$ in terms of $\overrightarrow{AB}$ and $\overrightarrow{AD}$. **(c)** Express $\overrightarrow{BD}$ in terms of $\overrightarrow{AB}$ and $\overrightarrow{AD}$.

331. Alex the geologist is in the desert again, 10 km from a long, straight road and 45 km from base camp. The base camp is also 10 km from the road, on the same side of the road as Alex is. On the road, the jeep can do 50 kph, but in the desert sands, it can manage only 30 kph. Alex wants to return to base camp as quickly as possible. How long will the trip take?

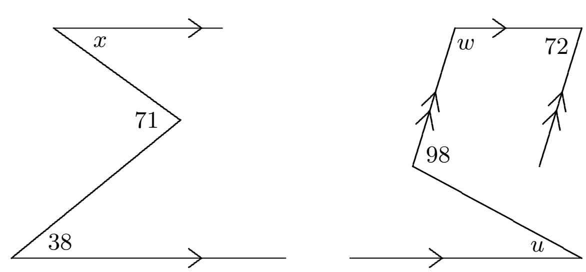


332. Find an equation that says that $P = (x, y)$ is equidistant from $F = (2, 0)$ and the y-axis. Plot four points that fit this equation. The configuration of all such points P is called a *parabola*.

333. Prove that the sum of the angles of any quadrilateral is 360 degrees. What about the sum of the angles of a pentagon? a hexagon? a 57-sided polygon?

334. Sketch the rectangular box that has one corner at $A = (0, 0, 0)$ and adjacent corners at $B = (12, 0, 0)$, $D = (0, 4, 0)$, and $E = (0, 0, 3)$. Find coordinates for G, the corner furthest from A. Find coordinates for P, the point on segment AG that is 5 units from A.

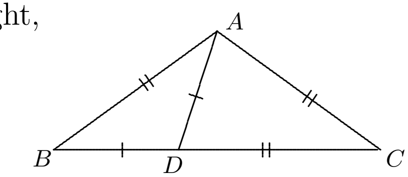
335. In the figures at right, find the sizes of the angles indicated by letters:



336. Given parallelogram $PQRS$, let T be the intersection of the bisectors of angles P and Q. Without knowing the sizes of the angles of $PQRS$, calculate the size of angle PTQ.

337. Let $F = (3, 2)$. There is a point P on the y-axis for which the distance from P to the x-axis equals the distance PF. Find the coordinates of P.

338. In the figure at right, it is given that BDC is straight, $BD = DA$, and $AB = AC = DC$. Find the size of angle C.



339. Mark the point P inside square $ABCD$ that makes triangle CDP equilateral. Calculate the size of angle PAD.

340. The converse of a statement of the form "If $\mathcal{A}$ then $\mathcal{B}$" is the statement "If $\mathcal{B}$ then $\mathcal{A}$."
(a) Write the converse of the statement "If point P is equidistant from the coordinate axes, then point P is on the line $y = x$."
(b) Give an example of a true statement whose converse is false.
(c) Give an example of a true statement whose converse is also true.

341. If a quadrilateral is a parallelogram, then both pairs of *opposite angles* are congruent. What is the converse of this statement? If the converse is a true statement, then prove it; if it is not, then explain why not.

342. By making a straight cut through one vertex of an isosceles triangle, Dylan dissected the triangle into two smaller isosceles triangles. Find the angle sizes of the original triangle. There is more than one possibility. How can you be sure that you have found them all?

343. In regular pentagon $ABCDE$, draw diagonal AC. What are the sizes of the angles of triangle ABC? Prove that segments AC and DE are parallel.

344. Given square $ABCD$, let P and Q be the points outside the square that make triangles CDP and BCQ equilateral. Prove that triangle APQ is also equilateral.

345. The sides of an equilateral triangle are 12 cm long. How long is an altitude of this triangle? What are the angles of a right triangle created by drawing an altitude? How does the short side of this right triangle compare with the other two sides?

346. If a quadrilateral is a parallelogram, then both pairs of opposite sides are congruent. Explain. What is the converse of this statement? Is it true?

347. In triangle ABC, it is given that angle A is 59 degrees and angle B is 53 degrees. The altitude from B to line AC is extended until it intersects the line through A that is parallel to segment BC; they meet at K. Calculate the size of angle AKB.

348. Given square $ABCD$, let P and Q be the points outside the square that make triangles CDP and BCQ equilateral. Segments AQ and BP intersect at T. Find angle ATP.

349. Give an example of a vector perpendicular to $[6, 2, 3]$ that has the same length.

350. Make an accurate drawing of an acute-angled, non-equilateral triangle ABC and its circumcenter K. Use your protractor to measure **(a)** angles A and BKC; **(b)** angles B and CKA; **(c)** angles C and AKB. Do you notice anything?

351. If the diagonals of a quadrilateral bisect each other, then the figure is a parallelogram. Prove that this is so. What about the converse statement?

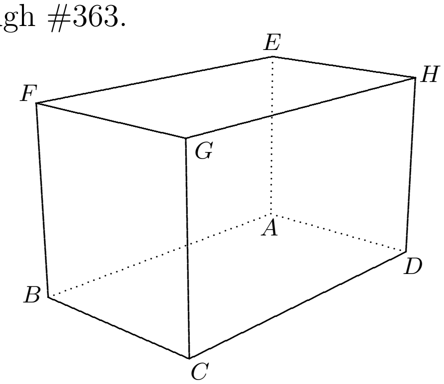
352. Suppose that one of the medians of a triangle happens to be exactly half the length of the side to which it is drawn. What can be said about the angles of this triangle? Justify your response.

353. (Continuation) Prove that the midpoint of the hypotenuse of a right triangle is equidistant from all three vertices of the triangle. How does this statement relate to the preceding?

354. Tate walks along the boundary of a four-sided plot of land, writing down the number of degrees turned at each corner. What is the sum of these four numbers? Would the answer be different if the plot of land were five-sided?

355. How can one tell whether a given quadrilateral is a parallelogram? In other words, how much evidence is needed to be sure of such a conclusion?

356. Two of the corners of rectangular box $ABCDEFGH$ are $A = (2, 1, 3)$ and $G = (9, 5, 7)$. Edges AB and AD are parallel to the x-and y-axes, respectively. Add the coordinate axes to the diagram, which also illustrates questions #357 through #363.



357. (Continuation) Find
(a) coordinates for the other six vertices;
(b) the lengths AH, AC, AF, FD, and AG;
(c) the distance from G to the xy-plane;
(d) the distance from G to the z-axis;
(e) what C, D, H, and G have in common.

358. (Continuation) Is angle FCH a right angle? Explain.

359. (Continuation) Find the areas of quadrilaterals $CDEF$ and $CAEG$.

360. (Continuation) Show that segments FD and CE bisect each other.

361. (Continuation) How many rectangles can be formed by joining four of the eight vertices?

362. (Continuation) A bug crawls linearly, with constant speed, from C to F, taking an hour for the trip. What are the coordinates of the bug after 24 minutes of crawling?

363. (Continuation) A fly flies linearly, with constant speed, from C to E, taking one minute for the flight. What are the coordinates of the fly after 24 seconds of flying?

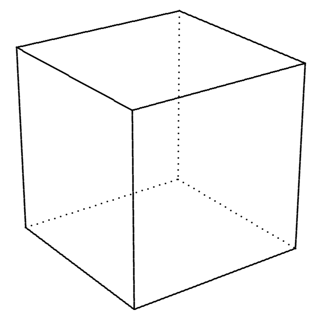
364. Marty walks along the boundary of a seventy-sided plot of land, writing down the number of degrees turned at each corner. What is the sum of these seventy numbers?

365. A right triangle has a 24-cm perimeter, and its hypotenuse is twice as long as its shorter leg. To the nearest tenth of a cm, find the lengths of all three sides of this triangle.

366. Preparing to go on a fishing trip to Alaska, Sam wants to know whether a collapsible fishing rod will fit into a rectangular box that measures 40 inches by 20 inches by 3 inches. The longest section of the rod is 44.75 inches long. Will the rod fit in the box?

367. Alex the geologist is in the desert, 10 km from a long, straight road. Alex's jeep does 50 kph on the road and 30 kph in the desert. Alex must return immediately to base camp, which is on the same side of the road, 10 km from the road, and d km from Alex. It so happens that the quickest possible trip will take the same amount of time, whether Alex uses the road or drives all the way in the desert. Find d.

368. Let $A = (0, 0)$, $B = (8, 0)$, and $C = (x, x)$. Find x, given that
(a) $BC = 7$; **(b)** $BC = 4\sqrt{2}$; **(c)** $BC = 10$ and CAB is a 45-degree angle.



369. Show that all the interior diagonals of a cube with 8-inch edges **(a)** have equal length, **(b)** bisect each other, and **(c)** are not perpendicular to each other.

370. Given $A = (7, 1, 3)$ and $C = (4, -2, 3)$, find the coordinates of the midpoint of segment AC.

371. (Continuation) Given $B = (5, 1, 2)$ and $D = (6, m, n)$, find m and n so that segments BD and AC have a common midpoint. Is $ABCD$ a parallelogram? Explain.

372. Questions #325, #354, and #364 illustrate the *Sentry Theorem*. What does this theorem say, and why has it been given this name?

373. *Midline Theorem.* Draw a triangle ABC, and let M and N be the midpoints of sides AB and AC, respectively. Express $\overrightarrow{BC}$ and $\overrightarrow{MN}$ in terms of $\mathbf{u} = \overrightarrow{AB}$ and $\mathbf{v} = \overrightarrow{AC}$.

374. Give coordinates for a point that is 8 units from the line $y = 5$. Then find both points on the line $3x + 2y = 4$ that are 8 units from the line $y = 5$.

375. Let $F = (0, 4)$. Find coordinates for three points that are equidistant from F and the x-axis. Write an equation that says that $P = (x, y)$ is equidistant from F and the x-axis.

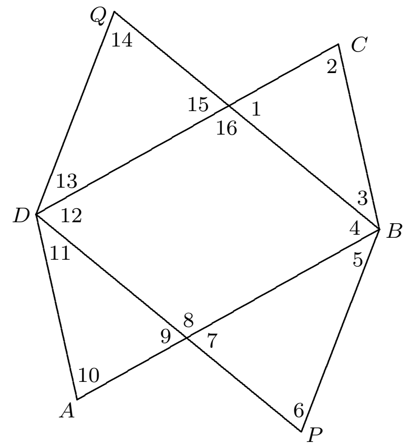
376. Given rectangle $ABCD$, let P be the point outside $ABCD$ that makes triangle CDP equilateral, and let Q be the point outside $ABCD$ that makes triangle BCQ equilateral. Prove that triangle APQ is also equilateral.

377. A regular, n-sided polygon has 18-degree exterior angles. Find the integer n.

378. Let $A = (0, 0)$, $B = (7, 2)$, $C = (3, 4)$, $D = (3, 7)$, and $E = (-1, 5)$. Cameron walks the polygonal path $ABCDEA$, writing down the number of degrees turned at each corner. What is the sum of these five numbers? Notice that $ABCDE$ is not a *convex* pentagon.

379. Draw a triangle ABC, and let AM and BN be two of its medians, which intersect at G. Extend AM to the point P that makes $GM = MP$. Prove that $PBGC$ is a parallelogram.

380. In the figure at right, it is given that $ABCD$ and $PBQD$ are parallelograms. Which of the numbered angles must be the same size as the angle numbered 1?



381. Given parallelogram $ABCD$, with diagonals AC and BD intersecting at O, let POQ be any line with P on AB and Q on CD. Prove that $AP = CQ$.

382. Triangle PQR has a right angle at P. Let M be the midpoint of QR, and let F be the point where the altitude through P meets QR. Given that angle FPM is 18 degrees, find the sizes of angles Q and R.

383. Given that $ABCDEFG\cdots$ is a regular n-sided polygon, in which angle $CAB = 12$ degrees, find n.

384. An Unidentified Flying Object (UFO) moving along a line with constant speed was sighted at $(8, 9, 10)$ at noon and at $(13, 19, 20)$ at 1:00 pm. Where was the UFO at 12:20 pm? When, and from where, did it leave the ground ($z = 0$)? What was the UFO's speed?

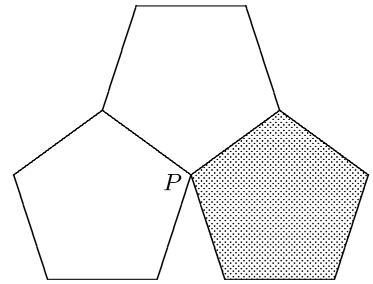
385. Find the image of the point (m, n) after it is reflected across the line $y = 2x$. After you are done, you should check your formulas. For example, they should confirm that $(4, 3)$ and $(0, 5)$ are images of each other. Your formulas should also confirm those points that are equal to their images — what points are these?

386. In triangle PQR, it is given that angle R measures r degrees. The bisectors of angles P and Q are drawn, creating two acute angles where they intersect. In terms of r, express the number of degrees in these acute angles.

387. Draw triangle ABC so that angles A and B are both 42 degrees. Why should AB be longer than BC? Extend CB to E, so that $CB = BE$. Mark D on AB so that $DB = BC$, then draw the line ED, which intersects AC at F. Find the size of angle CFD.

388. Draw a triangle ABC and two of its medians AM and BN. Let G be the point where AM intersects BN. Extend AM to the point P that makes $GM = MP$. Extend BN to the point Q that makes $GN = NQ$.
(a) Explain why BG must be parallel to PC, and AG must be parallel to QC.
(b) What kind of a quadrilateral is $PCQG$? How do you know?
(c) Find two segments in your diagram that *must* have the same length as BG.
(d) How do the lengths of segments BG and GN compare?

389. The diagram at right shows three congruent regular pentagons that share a common vertex P. The three polygons do not quite surround P. Find the size of the uncovered acute angle at P.



390. (Continuation) If the shaded pentagon were removed, it could be replaced by a regular n-sided polygon that would exactly fill the remaining space. Find the value of n that makes the three polygons fit perfectly.

391. You are given a square $ABCD$, and midpoints M and N are marked on BC and CD, respectively. Draw AM and BN, which meet at Q. Find the size of angle AQB.

392. Mark Y inside regular pentagon $PQRST$, so that PQY is equilateral. Is RYT straight? Explain.

393. An airplane that took off from its airport at noon ($t = 0$ hrs) moved according to the formula $(x, y, z) = (15, -20, 0) + t[450, -600, 20]$, where distance is in km. What is the meaning of the coordinate 0 in the equation? After twelve minutes, the airplane flew over Bethlehem, NH. Where is the airport in relation to Bethlehem, and how high was the airplane above the town?

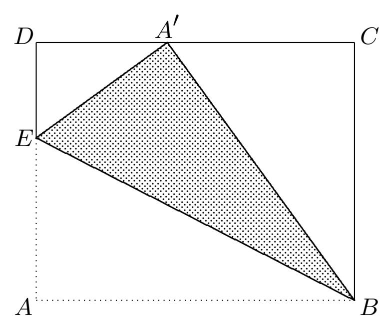
394. Suppose that triangle ABC has a right angle at B, that BF is the altitude drawn from B to AC, and that BN is the median drawn from B to AC. Find angles ANB and NBF, given that **(a)** angle C is 42 degrees; **(b)** angle C is 48 degrees.

395. Draw a parallelogram $ABCD$, then attach equilateral triangles CDP and BCQ to the outside of the figure. Decide whether or not triangle APQ is equilateral. Explain.

396. Suppose that $ABCD$ is a rhombus and that the bisector of angle BDC meets side BC at F. Prove that angle DFC is three times the size of angle FDC.

397. The midpoints of the sides of a triangle are $(3, -1)$, $(4, 3)$, and $(0, 5)$. Find coordinates for the vertices of the triangle.

398. In the diagram at right, a rectangular sheet of paper $ABCD$ has been creased so that corner A is now placed on edge CD, at A'. Find the size of angle DEA', given that the size of angle ABE is **(a)** 30 degrees; **(b)** 27 degrees; **(c)** n degrees.



399. Suppose that quadrilateral $ABCD$ has the property that AB and CD are congruent and parallel. Is this enough information to prove that $ABCD$ is a parallelogram? Explain.

400. Suppose that square $PQRS$ has 15-cm sides, and that G and H are on QR and PQ, respectively, so that PH and QG are both 8 cm long. Let T be the point where PG meets SH. Find the size of angle STG, with justification.

401. (Continuation) Find the lengths of PG and PT.

402. There are four special types of lines associated with triangles: Medians, perpendicular bisectors, altitudes, and angle bisectors.
(a) Which of these lines *must* go through the vertices of the triangle?
(b) Is it possible for a median to also be an altitude? Explain.
(c) Is it possible for an altitude to also be an angle bisector? Explain.

403. The diagonals of a rhombus have lengths 18 and 24. How long are the sides of the rhombus?

404. A *trapezoid* is a quadrilateral with exactly one pair of parallel sides. If the non-parallel sides have the same length, the trapezoid is *isosceles*. Make a diagram of an isosceles trapezoid whose sides have lengths 7 in, 10 in, 19 in, and 10 in. Find the *altitude* of this trapezoid (the distance that separates the parallel sides), then find the enclosed area.

405. Find three specific points that are equidistant from $F = (4, 0)$ and the line $y = x$.

406. Draw a triangle ABC and let N be the midpoint of segment AC. Express $\overrightarrow{BN}$ in terms of $\mathbf{u} = \overrightarrow{AB}$ and $\mathbf{v} = \overrightarrow{AC}$.

407. (Continuation) Let M be the midpoint of BC. Write $\overrightarrow{AM}$ in terms of $\mathbf{u}$ and $\mathbf{v}$.

408. (Continuation) Express $\overrightarrow{AB} + \frac{2}{3}\overrightarrow{BN}$ in terms of $\mathbf{u}$ and $\mathbf{v}$. Express $\frac{2}{3}\overrightarrow{AM}$ in terms of $\mathbf{u}$ and $\mathbf{v}$. Hmm...

409. If a quadrilateral is a rectangle, then its diagonals have the same length. What is the converse of this true statement? Is the converse true? Explain.

410. The diagonals of a parallelogram always bisect each other. Is it possible for the diagonals of a trapezoid to bisect each other? Explain.

411. A trapezoid has a 60-degree angle and a 45-degree angle. What are the other angles?

412. A trapezoid has a 60-degree angle and a 120-degree angle. What are the other angles?

413. The sides of a triangle have lengths 9, 12, and 15. (This is a special triangle!)
(a) Find the lengths of the medians of the triangle.
(b) The medians intersect at the centroid of the triangle. How far is the centroid from each of the vertices of the triangle?

414. (Continuation) Apply the same questions to the equilateral triangle of side 6.

415. Find the vertices and the area of the triangle formed by $y = |x - 3|$ and $-x + 2y = 5$.

416. Trapezoid $ABCD$ has parallel sides AB and CD, a right angle at D, and the lengths $AB = 15$, $BC = 10$, and $CD = 7$. Find the length DA.

417. Equilateral triangles BCP and CDQ are attached to the outside of regular pentagon $ABCDE$. Is quadrilateral $BPQD$ a parallelogram? Justify your answer.

418. A line of positive slope is drawn so that it makes a 60-degree angle where it intersects the x-axis. What is the slope of this line?

419. Mark P inside square $ABCD$, so that triangle ABP is equilateral. Let Q be the intersection of BP with diagonal AC. Triangle CPQ looks isosceles. Is this actually true?

420. What can be said about a quadrilateral, if it is known that every one of its adjacent-angle pairs is supplementary?

421. If $MNPQRSTUV$ is a regular polygon, then how large is each of its interior angles? If MN and QP are extended to meet at A, then how large is angle PAN?

422. Is it possible for the sides of a triangle to be 23, 19, and 44? Explain.

423. Suppose that $ABCD$ is a square, with $AB = 6$. Let N be the midpoint of CD and F be the intersection of AN and BD. What is the length of AF?

424. Prove that an isosceles trapezoid must have two pairs of equal adjacent angles.

425. (Continuation) The converse question: If a trapezoid has two pairs of equal adjacent angles, is it necessary that its non-parallel sides have the same length? Explain.

426. Let $F = (2, 3)$. Find coordinates for three points that are equidistant from F and the y-axis. Write an equation that says $P = (x, y)$ is equidistant from F and the y-axis.

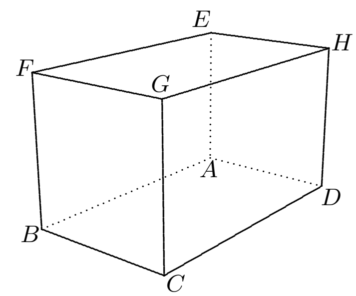
427. Write an equation for the line that is equidistant from $5x + 3y = 15$ and $5x + 3y = 27$.

428. The parallel sides of trapezoid $ABCD$ are AD and BC. Given that sides AB, BC, and CD are each half as long as side AD, find the size of angle D.

429. Squares $OPAL$ and $KEPT$ are attached to the outside of equilateral triangle PEA.
(a) Draw segment TO, then find the size of angle TOP.
(b) Decide whether segments EO and AK have the same length, and give your reasons.

430. The lengths of the sides of triangle ABC are $AB = 15 = AC$ and $BC = 18$. Find the distance from A to **(a)** the centroid of ABC; **(b)** the circumcenter of ABC.

431. The diagram shows rectangular box $ABCDEFGH$, with $A = (2, 3, 1)$, $G = (10, 7, 5)$, and edges parallel to the coordinate axes.



(a) Write parametric equations for line FD.
(b) Let M be the midpoint of segment CD and draw segment EM. Find coordinates for the point P that is two thirds of the way from E to M.
(c) Show that P is also on segment FD. Why was it predictable that segments FD and EM would intersect?

432. A parallelogram has two 19-inch sides and two 23-inch sides. What is the range of possible lengths for the diagonals of this parallelogram?

433. The altitudes of an equilateral triangle all have length 12 cm. How long are its sides?

434. It is given that the sides of an isosceles trapezoid have lengths 3 in, 15 in, 21 in, and 15 in. Make a diagram. Show that the diagonals intersect perpendicularly.

435. Triangle ABC has $AB = AC$. The bisector of angle B meets AC at D. Extend side BC to E so that $CE = CD$. Triangle BDE should look isosceles. Is it? Explain.

436. If $ABCD$ is a quadrilateral, and BD bisects both angle ABC and angle CDA, then what sort of quadrilateral must $ABCD$ be?

437. In quadrilateral $ABCD$, angles ABC and CDA are both bisected by BD, and angles DAB and BCD are both bisected by AC. What sort of quadrilateral must $ABCD$ be?

438. Given a triangle, you have proved the following result: The point where two medians intersect (the *centroid*) is twice as far from one end of a median as it is from the other end of the same median. Improve the statement of the preceding theorem so that the reader knows which end of the median is which. This theorem indirectly shows that the three medians of any triangle must be *concurrent*. Explain the reasoning.

439. Find coordinates for the centroid of the triangle whose vertices are
(a) $(2, 7)$, $(8, 1)$, and $(14, 11)$; **(b)** (a, p), (b, q), and (c, r).

440. Let $ABCD$ be a parallelogram, with M the midpoint of DA, and diagonal AC of length 36. Let G be the intersection of MB and AC. What is the length of AG?

441. The diagonals of a square have length 10. How long are the sides of the square?

442. Triangle PQR is isosceles, with $PQ = 13 = PR$ and $QR = 10$. Find the distance from P to the centroid of PQR. Find the distance from Q to the centroid of PQR.

443. (Continuation) Find the distance from P to the circumcenter of triangle PQR.

444. For what triangles is it true that the circumcenter and the centroid are the same point?

445. Is it possible for the diagonals of a parallelogram to have the same length? How about the diagonals of a trapezoid? How about the diagonals of a non-isosceles trapezoid?

446. Calculate the effect of the transformation $\mathcal{T}(x, y) = \left(\frac{3}{5}\,x + \frac{4}{5}\,y\,,\ \frac{4}{5}\,x - \frac{3}{5}\,y\right)$ on a triangle of your choosing. Is $\mathcal{T}$ an isometry? If so, what kind?

447. How many diagonals can be drawn inside a pentagon? a hexagon? a decagon? a twenty-sided polygon? an n-sided polygon?

448. Suppose that P is twice as far from $(0, 0)$ as P is from $(6, 0)$. Find such a point on the x-axis. Find another such point that is *not* on the x-axis.

449. Let $A = (1.43, 10.91)$, $B = (3.77, 7.33)$, and $C = (8.15, 2.55)$. Find coordinates for G, the centroid of triangle ABC. Find an equation for the line through G parallel to AC.

450. In triangle ABC, let M be the midpoint of AB and N be the midpoint of AC. Suppose that you measure MN and find it to be 7.3 cm long. How long would BC be, if you measured it? If you were to measure angles AMN and ABC, what would you find?

451. In triangle TOM, let P and Q be the midpoints of segments TO and TM, respectively. Draw the line through P parallel to segment TM, and the line through Q parallel to segment TO; these lines intersect at J. What can you say about the location of point J?

452. Let G be the centroid of triangle ABC. Simplify the vector sum $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC}$.

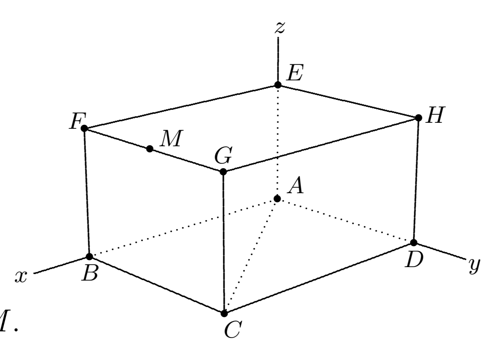
453. Mark $A = (0, 0)$ and $B = (10, 0)$ on your graph paper, and use your protractor to draw the line of positive slope through A that makes a 25-degree angle with AB. By making suitable measurements, calculate (approximately) the slope of this line.

454. (Continuation) With your calculator in degree mode, find a decimal approximation for the expression $\tan(25)$.

455. (Continuation) Repeat the preceding construction and tan verification for several more angles of your choosing.

456. Find coordinates for a point that is 5 units from the line $3x + 4y = 10$.

457. The diagram at right shows rectangular box $ABCDEFGH$, with $A = (0, 0, 0)$, $G = (4, 3, 2)$, and the sides parallel to the coordinate axes. The midpoint of FG is M.



(a) Find coordinates for M.
(b) Find coordinates for the point P on segment AC that is 2 units from A.
(c) Decide whether angle APM is a right angle, and give your reasons.
(d) Find the point on segment AC that is closest to M.

458. Find coordinates for the centroid of the triangle whose vertices are $(2, 1, 3)$, $(4, 5, 6)$, and $(0, 3, 1)$.

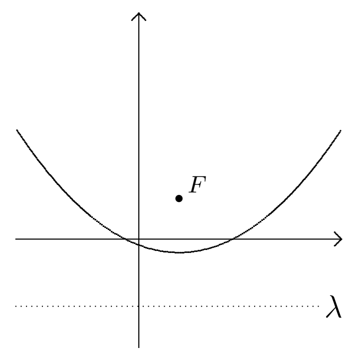
459. Let $A = (0, 0)$, $B = (12, 4)$, $C = (2, 3)$, $D = (8, 5)$, $E = (5, -3)$, and $F = (11, -1)$. Draw lines AB, CD, and EF. Verify that they are parallel.
(a) Draw the transversal of slope -1 that goes through F. This transversal intersects line AB at G and line CD at H. Use your ruler to measure FG and GH.
(b) Draw any transversal whose slope is 3, and let P, Q, and R be its intersections with lines AB, CD, and EF, respectively. Use your ruler to measure PQ and PR.
(c) On the basis of your findings, formulate the *Three Parallels Theorem*.

460. A line drawn parallel to the side BC of triangle ABC intersects side AB at P and side AC at Q. The measurements $AP = 3.8$ in, $PB = 7.6$ in, and $AQ = 5.6$ in are made. If segment QC were now measured, how long would it be?

461. Draw an acute-angled triangle ABC, and mark points P and Q on sides AB and AC, respectively, so that $AB = 3AP$ and $AC = 3AQ$. Express $\overrightarrow{PQ}$ and $\overrightarrow{BC}$ in terms of $\mathbf{v} = \overrightarrow{AP}$ and $\mathbf{w} = \overrightarrow{AQ}$.

462. Let $A = (7, 1, 1)$ and $B = (-3, 2, 7)$. Find all the points P on the z-axis that make angle APB right.

463. Let $F = (3, 3)$. The points $P = (x, y)$ that are equidistant from F and the line $y = -5$ form a parabola.
(a) Find the points where this parabola meets the x-axis.
(b) Find the coordinates of the *vertex*.



464. (Continuation) Find an equation for the parabola.

465. Given a line λ (*Greek* "lambda") and a point F not on λ, let P be on the parabola of all points that are equidistant from F and from λ. Let N be the point on λ closest to P. Explain why any other point Q on the parabola must be closer to F than to N. What does this tell you about the perpendicular bisector of FN?

466. Is it possible for a scalene triangle to have two medians of the same length? Explain.

467. Standing 50 meters from the base of a fir tree, Rory used a protractor to measure an *angle of elevation* of 33° to the top of the tree. How tall was the tree?

468. Given $A = (0, 6)$, $B = (-8, 0)$, and $C = (8, 0)$, find the circumcenter of triangle ABC.

469. Given regular hexagon $BAGELS$, show that SEA is an equilateral triangle.

470. Let $A = (0, 0)$, $B = (0, 3)$, and $C = (4, 0)$. Let F be the point where the bisector of angle BAC meets side BC. Find exact coordinates for F. Notice that F is not the midpoint of BC. Finally, calculate the distances BF and CF. Do you notice anything?

471. Choose four lattice points for the vertices of a non-isosceles trapezoid $ABCD$, with AB longer than CD and parallel to CD. Extend AD and BC until they meet at E. Verify that the ratios $\dfrac{AD}{DE}$ and $\dfrac{BC}{CE}$ are equal, by measurement or calculation.

472. Segments AC and BD intersect at E, so as to make AE twice EC and BE twice ED. Prove that segment AB is twice as long as segment CD, and parallel to it.

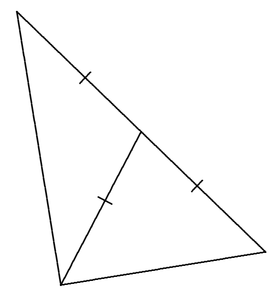
473. When the sun has risen 32 degrees above the horizon, a *lower* casts a shadow that is 9 feet 2 inches long. How tall is the lower, to the nearest inch?

474. In the following list of true statements, find **(a)** the statements whose converses are also in the list; **(b)** the statement that is a definition; **(c)** a statement whose converse is false; **(d)** the Sentry Theorem; and **(e)** the Midline Theorem.

1. If a quadrilateral has two pairs of parallel sides, then its diagonals bisect each other.

2. If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral must be a parallelogram.

3. If a quadrilateral is equilateral, then it is a rhombus.



4. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

5. If a quadrilateral has two pairs of equal adjacent sides, then its diagonals are perpendicular.

6. If one of the medians of a triangle is half the length of the side to which it is drawn, then the triangle is a right triangle.

7. If a segment joins two of the midpoints of the sides of a triangle, then it is parallel to the third side, and is half the length of the third side.

8. Both pairs of opposite sides of a parallelogram are congruent.

9. The sum of the exterior angles of any polygon — one at each vertex — is 360 degrees.

10. The median drawn to the hypotenuse of a right triangle is half the length of the hypotenuse.

11. If two lines are intersected by a transversal so that alternate interior angles are equal, then the lines must be parallel.

12. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is in fact a parallelogram.

13. If two opposite sides of a quadrilateral are both parallel and equal in length, then the quadrilateral is a parallelogram.

14. If three parallel lines intercept equal segments on one transversal, then they intercept equal segments on every transversal.

15. Both pairs of opposite angles of a parallelogram are congruent.

16. The medians of any triangle are concurrent, at a point that is two thirds of the way from any vertex to the midpoint of the opposite side.

17. An exterior angle of a triangle is the sum of the two nonadjacent interior angles.

475. The position of an airplane that is approaching its airport is described parametrically by $P_t = (1000, 500, 900) + t[-100, -50, -90]$. For what value of t is the airplane closest to the traffic control center located at (34,68,16)?

476. Triangle ABC has $AB = 10 = AC$ and $BC = 12$. Find the distance from A to
(a) the centroid of ABC; **(b)** the circumcenter of ABC.

477. Suppose that a quadrilateral is measured and found to have a pair of equal nonadjacent sides and a pair of equal nonadjacent angles. Is this enough evidence to conclude that the quadrilateral is a parallelogram? Explain.

478. Let $P = (-15, 0)$, $Q = (5, 0)$, $R = (8, 21)$, and $S = (0, 15)$. Draw quadrilateral $PQRS$ and measure its sides and angles. Is there anything remarkable about this figure?

479. Given triangle ABC, let F be the point where segment BC meets the bisector of angle BAC. Draw the line through B that is parallel to segment AF, and let E be the point where this parallel meets the extension of segment CA.
(a) Find the four congruent angles in your diagram.
(b) How are the lengths EA, AC, BF, and FC related?
(c) *The Angle-Bisector Theorem*: How are the lengths AB, AC, BF, and FC related?

480. Standing on a cliff 380 meters above the sea, Pat sees an approaching ship and measures its *angle of depression*, obtaining 9 degrees. How far from shore is the ship?

481. (Continuation) Now Pat sights a second ship beyond the first. The angle of depression of the second ship is 5 degrees. How far apart are the ships?

482. Let $RICK$ be a parallelogram, with M the midpoint of RI. Draw the line through R that is parallel to MC; it meets the extension of IC at P. Prove that $CP = KR$.

483. Suppose that $ABCD$ is a trapezoid, with AB parallel to CD. Let M and N be the midpoints of DA and BC, respectively. What can be said about segment MN? Explain.

484. What is the radius of the smallest circle that encloses an equilateral triangle with 12-inch sides? What is the radius of the largest circle that will fit inside the same triangle?

485. In acute triangle ABC, the bisector of angle ABC meets side AC at D. Mark points P and Q on sides BA and BC, respectively, so that segment DP is perpendicular to BA and segment DQ is perpendicular to BC. Prove that triangles BDP and BDQ are congruent. What about triangles PAD and QCD?

486. Let $A = (0, 0)$, $B = (4, 0)$, and $C = (4, 3)$. Measure angle CAB. What is the slope of AC? Use this slope and the tan function to check your angle measurement. Use a calculator to come as close as you can to the theoretically correct size of angle CAB.

487. (Continuation) On a calculator, enter the expression $\tan^{-1}(0.75)$. Hmm ...

488. Find coordinates for the point P where the line $y = x$ intersects the line $2x + 3y = 24$. Then calculate the distances from P to the axis intercepts of $2x + 3y = 24$. The Angle-Bisector Theorem makes a prediction about these distances — what is the prediction?

489. A five-foot *upper* casts an eight-foot shadow. How high is the sun in the sky? This question is not asking for a distance, by the way.

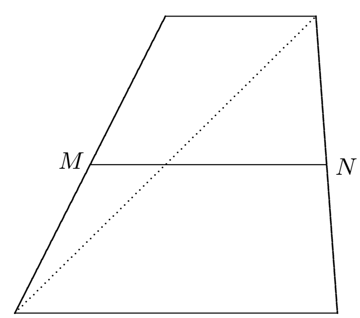
490. Are the points $(2, 5, 7)$, $(12, 25, 37)$, and $(27, 55, 81)$ collinear?

491. Find coordinates for the point where the line $(x, y, z) = (7 + 2r, 5 - 3r, 4 + r)$ intersects the xz-plane.

492. Inside regular pentagon $SOULE$ is marked point P so that triangle SOP is equilateral. Decide whether or not quadrilateral $SOUP$ is a parallelogram, and give your reasons.

493. Suppose that $ABCD$ is a parallelogram, in which $AB = 2BC$. Let M be the midpoint of segment AB. Prove that segments CM and DM bisect angles BCD and CDA, respectively. What is the size of angle CMD? Justify your response.

494. If M and N are the midpoints of the non-parallel sides of a trapezoid, it makes sense to call the segment MN the *mid-line* of the trapezoid. Why? (It actually should be called the *midsegment*, of course. Strange to say, some textbooks call it the *median*). Suppose that the parallel sides of a trapezoid have lengths 7 and 15. What is the length of the midline of the trapezoid? Notice that the midline is divided into two pieces by a diagonal of the trapezoid. What are the lengths of these pieces? Does it matter which diagonal is drawn?



495. An isosceles trapezoid has sides of lengths 9, 10, 21, and 10. Find the distance that separates the parallel sides, then find the length of the diagonals. Finally, find the angles of the trapezoid, to the nearest tenth of a degree.

496. Find the angle formed by the diagonal of a cube and a diagonal of a face of the cube.

497. One day at the beach, Kelly flies a kite, whose string makes a 37-degree elevation angle with the ground. Kelly is 130 feet from the point directly below the kite. How high above the ground is the kite, to the nearest foot?

498. Think of the ground you are standing on as the xy-plane. The vector $[12, 5, 10]$ points from you toward the sun. How high is the sun in the sky?

499. Hexagon $ABCDEF$ is regular. Prove that segments AE and ED are perpendicular.

500. Suppose that $PQRS$ is a rhombus, with $PQ = 12$ and a 60-degree angle at Q. How long are the diagonals PR and QS?

501. A triangle, whose sides are 6, 8, and 10, and a circle, whose radius is r, are drawn so that no part of the triangle lies outside the circle. How small can r be?

502. Diagonals AC and BD of regular pentagon $ABCDE$ intersect at H. Decide whether or not $AHDE$ is a rhombus, and give your reasons.

503. Do the lines $(x, y, z) = (5 + 2t, 3 + 2t, 1 - t)$ and $(x, y, z) = (13 - 3r, 13 - 4r, 4 - 2r)$ intersect? If so, at what point? If not, how do you know? Why are different parameters used?

504. Let $A = (3, 1)$, $B = (9, 5)$, and $C = (4, 6)$. Your protractor should tell you that angle CAB is about 45 degrees. Explain why angle CAB is in fact exactly 45 degrees.

505. Draw a regular pentagon and all five of its diagonals. How many isosceles triangles can you find in your picture? How many scalene triangles can you find?

506. The sides of a triangle are 6 cm, 8 cm, and 10 cm long. Find the distances from the centroid of this triangle to the three vertices.

507. The diagonals of a non-isosceles trapezoid divide the midline into three segments, whose lengths are 8 cm, 3 cm, and 8 cm. How long are the parallel sides? From this information, is it possible to infer anything about the distance that separates the parallel sides? Explain.

508. The sides of a polygon are cyclically extended to form *rays*, creating one exterior angle at each vertex. Viewed from a great distance, what theorem does this figure illustrate?

509. Given a rectangular card that is 5 inches long and 3 inches wide, what does it mean for another rectangular card to have the *same shape*? Describe a couple of examples.

510. A rectangle is 2 inches wide, and more than 2 inches long. It so happens that this rectangle can be divided, by a single cut, into a 2-inch square and a small rectangle that has exactly the same shape as the large rectangle. What is the length of the large rectangle?

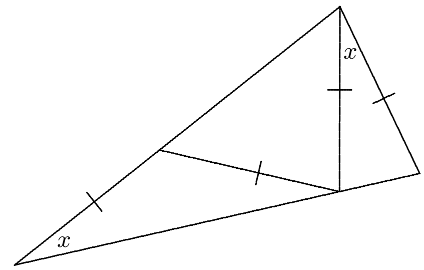
511. Let $A = (1, 2, 3)$, $B = (3, 7, 9)$, and $D = (-2, 3, -1)$. Find coordinates for vertex C of parallelogram $ABCD$. How many parallelograms can you find that have the three given vertices among their four vertices?

512. *The Orthocenter*. Given an acute-angled triangle ABC, draw the line through A parallel to BC, the line through B parallel to AC, and the line through C parallel to AB. These lines form triangle PQR. The altitudes of triangle ABC are also special lines for triangle PQR. Explain.

513. In trapezoid $ABCD$, AB is parallel to CD, and $AB = 10$, $BC = 9$, $CD = 22$, and $DA = 15$. Points P and Q are marked on BC so that $BP = PQ = QC = 3$, and points R and S are marked on DA so that $DR = RS = SA = 5$. Find the lengths PS and QR.

514. *The Varignon quadrilateral.* A quadrilateral has diagonals of lengths 8 and 10. The midpoints of the sides of this figure are joined to form a new quadrilateral. What is the perimeter of the new quadrilateral? What is special about it?

515. In the figure at right, there are two x-degree angles, and four of the segments are congruent as marked. Find x.



516. The parallel bases of a trapezoid have lengths 12 and 18 cm. Find the lengths of the two segments into which the midline of the trapezoid is divided by a diagonal.

517. In triangle ABC, points M and N are marked on sides AB and AC, respectively, so that $AM : AB = 17 : 100 = AN : AC$. Show that segments MN and BC are parallel.

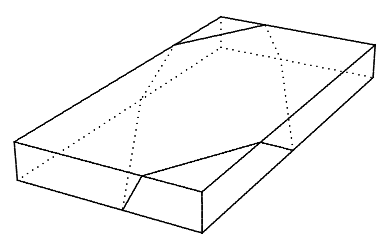
518. (Continuation) In triangle ABC, points M and N are marked on sides AB and AC, respectively, so that the ratios $AM : AB$ and $AN : AC$ are both equal to r, where r is some number between 0 and 1. Show that segments MN and BC are parallel.

519. A cheetah can run at 105 feet per second, but only for 7 seconds, at which time the animal must stop and rest. A fully rested cheetah at $(0, 0)$ notices a nearby antelope, which is moving according to the parametric equation $(x, y) = (-39 + 40t, 228 + 30t)$, where t is measured in seconds and x and y are measured in feet. If it started to run at $t = 0$, the cheetah could catch the antelope. For how many more seconds can the cheetah afford to wait before starting? Assume that the cheetah does not change direction when it runs.

520. The diagonals of rhombus $ABCD$ meet at M. Angle DAB measures 60 degrees. Let P be the midpoint of AD, and let G be the intersection of PC and MD. Given that $AP = 8$, find MD, MC, MG, CG, and GP.

521. Rectangle $ABCD$ has dimensions $AB = 5$ and $BC = 12$. Let M be the midpoint of BC, and let G be the intersection of AM and diagonal BD. Find BG and AG.

522. The hypotenuse of a right triangle is three times the length of the shortest leg. What is the ratio of the larger leg to the smaller leg? What is the size of the smallest angle?



523. What is the smallest amount of ribbon that is needed to wrap around a $2'' \times 10'' \times 20''$ gift box in the way shown in the figure at right? You could experiment with some string and a book.

524. What are the angle sizes in an isosceles trapezoid with side lengths 6, 20, 6, and 26?

525. Given square $ABCD$, choose a point O that is not outside the square and form the vector $\mathbf{v} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD}$. By trying various possible positions for O, find the shortest and longest possible $\mathbf{v}$.

526. Find the angle formed by two face diagonals that intersect at a vertex of the cube.

527. Let $A = (0, 0)$, $B = (4, -3)$, $C = (6, 3)$, $P = (-2, 7)$, $Q = (9, 5)$, and $R = (7, 19)$. Measure the angles of triangles ABC and PQR. Calculate the lengths of the sides of these triangles. Find justification for any conclusions you make.

528. Find components for the vector that points from A to B when
(a) $A = (2, 5, -6)$ and $B = (x, y, z)$; **(b)** $A = (4, 1 + 2t, 3 - t)$ and $B = (1, 5, 0)$.

529. Find coordinates for a point that is 7 units from the line $4x + y = 11$.

530. While the Wood's Hole-Martha's Vineyard ferryboat steamed along at 8 mph through calm seas, passenger Casey exercised by walking the perimeter of the rectangular deck, at a steady 4 mph. Discuss the variations in Casey's speed *relative to the water*.

531. Brett and Jordan are cruising, according to the equations $B_t = (27+4t, 68-7t, 70+4t)$ and $J_t = (23 + 4t, 11 + t, 34 + 8t)$, where t represents elapsed time. Show that their paths intersect, but that there is no collision. To answer this, you will need to make a parameter change; why? Who reaches the intersection first? Who is moving faster?

532. Let $A = (4, 0, 0)$, $B = (0, 3, 0)$, and $C = (0, 0, 5)$.
(a) Draw a diagram, then find coordinates for the point closest to C on segment AB.
(b) Find the area of triangle ABC.
(c) Find the length of the altitude drawn from A to BC.

533. In triangle ABC, it is given that $AB = 4$, $AC = 6$, and $BC = 5$. The bisector of angle BAC meets BC at D. Find lengths BD and CD.

534. Let $A = (1, 2)$, $B = (8, 2)$, and $C = (7, 10)$. Find an equation for the line that bisects angle BAC.

535. Atiba wants to measure the width of the Squamscott River. Standing under a tree T on the river bank, Atiba sights a rock at the nearest point R on the opposite bank. Then Atiba walks to a point P on the river bank that is 50.0 meters from T, and makes RTP a right angle. Atiba then measures RPT and obtains 76.8 degrees. How wide is the river?

536. Let P be the circumcenter and G be the centroid of a triangle formed by placing two perpendicular vectors $\mathbf{v}$ and $\mathbf{w}$ tail to tail. Express $\overrightarrow{GP}$ in terms of vectors $\mathbf{v}$ and $\mathbf{w}$.

537. Out for a walk in Chicago, Morgan measures the angle of elevation to the distant Willis Tower, and gets 3.6 degrees. After walking one km directly toward the building, Morgan finds that the angle of elevation has increased to 4.2 degrees. Use this information to calculate the height of the Willis Tower, and how far Morgan is from it now.

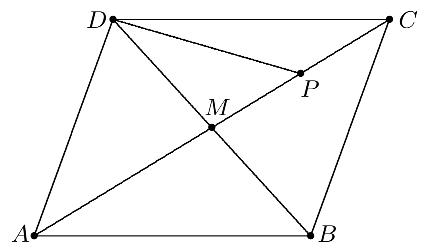
538. How tall is an isosceles triangle, given that its base is 30 cm long and that both of its base angles are 72 degrees?

539. Alex the geologist is in the desert, 18 km from a long, straight road and 72 km from base camp, which is also 18 km from the road, on the same side of the road as Alex is. On the road, the jeep can do 60 kph, but in the desert sands, it can manage only 32 kph.
(a) Describe the path that Alex should follow, to return to base camp most quickly.
(b) If the jeep were capable of 40 kph in the desert, how would your answer be affected?

540. Triangle ABC has $AB = 12 = AC$ and angle A is 120 degrees. Let F and D be the midpoints of sides AC and BC, respectively, and G be the intersection of segments AD and BF. Find the lengths FD, AD, AG, BG, and BF.

541. Simplify the vector expression $\overrightarrow{AB} - \overrightarrow{AC}$, and illustrate with a diagram.

542. Find the side of the largest square that can be drawn inside a 12-inch equilateral triangle, one side of the square aligned with one side of the triangle.



543. In the figure at right, $ABCD$ is a parallelogram, with diagonals AC and BD intersecting at M, and P the midpoint of CM. Express the following in terms of $\mathbf{u} = \overrightarrow{AB}$ and $\mathbf{v} = \overrightarrow{AD}$: **(a)** $\overrightarrow{AM}$ **(b)** $\overrightarrow{BD}$ **(c)** $\overrightarrow{CP}$ **(d)** $\overrightarrow{DP}$

544. Rectangle $ABCD$ has $AB = 16$ and $BC = 6$. Let M be the midpoint of side AD and N be the midpoint of side CD. Segments CM and AN intersect at G. Find the length AG.

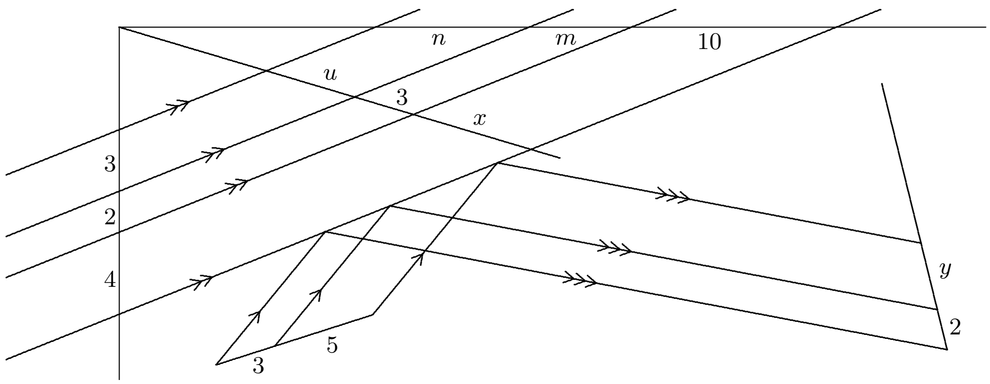
545. The parallel sides of a trapezoid have lengths m and n. The diagonals of the trapezoid divide the midline into three pieces. In terms of m and n, how long are the pieces?

546. To the nearest tenth of a degree, find the sizes of the acute angles in the 5-12-13 triangle and in the 9-12-15 triangle. This enables you to calculate the sizes of the angles in the 13-14-15 triangle. Show how to do it, then invent another example of this sort.

547. A triangle has a 60° angle and a 45° angle, and the side opposite the 45° angle is 240 mm long. Determine the exact length of the side opposite the 60° angle.

548. Explain how two congruent trapezoids can be combined without overlapping to form a parallelogram. What does this tell you about the length of the midline of the trapezoid?

549. In both of the figures below, find the lengths of the segments indicated by letters. Parallel lines are indicated by arrows.



550. An animal rescue shelter has 175 total cats and dogs. The ratio of cats to dogs is 3:2.
(a) How many cats are there?
(b) If 15 animals are selected at random to get their vaccinations, how many would you expect to be dogs?

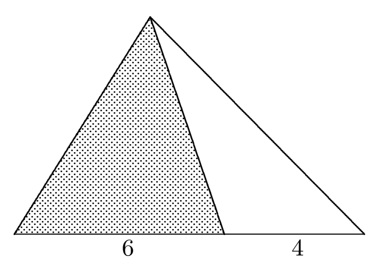
551. Rory, Leo, and Joey win \$362 880 in a lottery. They decide to divide the winnings so that Rory gets two parts, Leo three parts, and Joey four parts. Exactly how much money does each person receive?

552. Given that P is three fifths of the way from A to B, and that Q is one third of the way from P to B, describe the location of Q in relation to A and B.

553. Suppose that the points A, P, Q, and B appear in this order on a line, such that $AP : AB = 3 : 5$ and $PQ : QB = 1 : 2$. Evaluate the ratios $AQ : AB$ and $AQ : QB$.

554. Apply the transformation $\mathcal{T}(x, y) = (3x, 3y)$ to the triangle PQR whose vertices are $P = (3, -1)$, $Q = (1, 2)$, and $R = (4, 3)$. Compare the sides and angles of the image triangle $P'Q'R'$ with the corresponding parts of PQR. This transformation is an example of a *dilation*, also called a *radial expansion*. Is $\mathcal{T}$ an isometry?

555. In the figure at right, the shaded triangle has area 15. Find the area of the unshaded triangle.



556. In a triangle whose sides have lengths 3, 4, and 5,
(a) how long is the bisector of the larger of the two acute angles?
(b) how long is the bisector of the right angle?

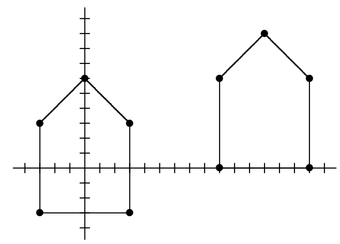
557. Show that the altitude drawn to the hypotenuse of any right triangle divides the triangle into two triangles that have the same angles as the original.

558. Compare the quadrilateral whose vertices are $A = (0, 0)$, $B = (6, 2)$, $C = (5, 5)$, $D = (-1, 3)$ with the quadrilateral whose vertices are $P = (9, 0)$, $Q = (9, 2)$, $R = (8, 2)$, and $S = (8, 0)$. Calculate lengths and angles, and look for patterns.

559. On a piece of graph paper, draw a right triangle that has a 15-unit hypotenuse and a $27°$ angle. To the nearest tenth of a unit, measure the side opposite the $27°$ angle, and then express your answer as a fraction of the length of the hypotenuse. Repeat this process with a right triangle that has a 10-unit hypotenuse and a $27°$ angle. Compare your answers with the value obtained from a calculator when you enter $\sin(27)$ in degree mode.

560. (Continuation) Repeat the process twice with two different angles of your choosing. Write a summary of your findings.

561. Apply $\mathcal{T}(x, y) = (2x/3\,, 2y/3)$ to the following pentagons:
(a) vertices $(3, -3)$, $(3, 3)$, $(0, 6)$, $(-3, 3)$, and $(-3, -3)$;
(b) vertices $(15, 0)$, $(15, 6)$, $(12, 9)$, $(9, 6)$, and $(9, 0)$.
Are the results what you expected?



562. The area of a parallelogram can be found by multiplying the distance between two parallel sides by the length of either of those sides. Explain why this formula works.

563. The area of a trapezoid can be found by multiplying its altitude (the distance between the parallel sides) by the length of its midline. Explain why this formula works. One approach is to find a suitable rectangle that has the same area as the trapezoid.

564. The parallel sides of a trapezoid have lengths 9 cm and 12 cm. Draw one diagonal, dividing the trapezoid into two triangles. What is the ratio of their areas? If the other diagonal had been drawn instead, would this have affected your answer?

565. If triangle ABC has a right angle at C, the ratio $AC : AB$ is called the *sine ratio of angle B*, or simply the *sine of B*, and is usually written *sin B*. What should the ratio $BC : AB$ be called? Determine by hand the sine ratio for a 30-degree angle and for a 60-degree angle.

566. Find a vector of length 3 that is perpendicular to **(a)** $[2, 1, -2]$; **(b)** $[4, 4, 7]$.

567. Let $C = (1, 4)$, $P = (5, 2)$, and $P' = (13, -2)$. There is a dilation that leaves C where it is and transforms P into P'. The point C is called the *dilation center*. Explain why the *magnitude* of this dilation is 3. Determine the coordinates of Q', given that $Q = (3, 5)$. Determine the coordinates of R, given that $R' = (-6, 7)$.

568. Given $A = (4, 1, 3)$ and $B = (6, 2, 1)$, find coordinates for points C and D that make $ABCD$ a square. There are many possible answers.

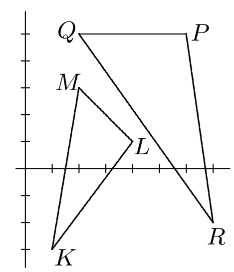
569. Apply the transformation $\mathcal{T}(x, y) = (0.8x - 0.6y, 0.6x + 0.8y)$ to the scalene triangle whose vertices are $(0, 0)$, $(5, 0)$, and $(0, 10)$. What kind of isometry does $\mathcal{T}$ seem to be? Be as specific as you can, and provide numerical evidence for your conclusion.

570. Consider the dilation $\mathcal{T}(x, y) = (mx, my)$, where m is a positive number. If $m > 1$, then $\mathcal{T}$ is a radial expansion; if $m < 1$, then $\mathcal{T}$ is a radial contraction. Regardless of the value of m, show that $\mathcal{T}$ transforms any segment with endpoints $P = (a, b)$ and $Q = (c, d)$ onto an image segment that is parallel and m times as long. Hint: vectors might prove easier than the Pythagorean Theorem.

571. To draw a right triangle that has a 1° angle and measure its sides accurately is difficult. To get the sine ratio for a 1° angle, however, there is an easy way — just use a calculator. Is the ratio a small or large number? How large can a sine ratio be?

572. On a piece of graph paper, draw a right triangle that has an 18-unit hypotenuse and a 70° angle. To within 0.1 unit, measure the leg *adjacent* to the 70° angle, and express your answer as a fraction of the hypotenuse. Compare your answer with the value obtained from a calculator when you enter cos (70) in degree mode. This is an example of the *cosine ratio*.

573. One figure is *similar* to a second figure if the points of the first figure can be matched with the points of the second figure in such a way that corresponding distances are proportional. In other words, there is a number m — the *ratio of similarity* — with the following property: If A and B are any two points whatsoever of the first figure, and A' and B' are their corresponding images in the second figure, then the distance from A' to B' is m times the distance from A to B.



The triangle defined by $K = (1, -3)$, $L = (4, 1)$, and $M = (2, 3)$ is similar to the triangle defined by $P = (6, 5)$, $Q = (2, 5)$, and $R = (7, -2)$. Confirm the proportionality of lengths for *four* segments (include a non-edge) and their images.

574. Show that any dilation transforms any figure into a similar figure.

575. Given two similar figures, it might not be possible to transform one into the other using only a dilation. Explain this remark, using the triangles KLM and RPQ shown above.

576. (Continuation) A carefully chosen rotation followed by a carefully chosen dilation can be used to transform triangle KLM into triangle RPQ. Explain this remark.

577. A rhombus has four 6-inch sides and two 120° angles. From one of the vertices of the obtuse angles, the two altitudes are drawn, dividing the rhombus into three pieces. Find the areas of these pieces.

578. When triangle ABC is similar to triangle PQR, with A, B, and C corresponding to P, Q, and R, respectively, it is customary to write $\triangle ABC \sim \triangle PQR$. Suppose that $AB = 4$, $BC = 5$, $CA = 6$, and $RP = 9$. Find PQ and QR.

579. What is the size of the acute angle formed by the x-axis and the line $3x + 2y = 12$?

580. To the nearest tenth of a degree, find the sizes of the acute angles in the right triangle whose long leg is 2.5 times as long as its short leg.

581. Let $A = (0, 0)$, $B = (15, 0)$, $C = (5, 8)$, $D = (9, 0)$, and $P = (6, 6)$. Draw triangle ABC, segments CD, PA, and PB, and notice that P is on segment CD. There are now three pairs of triangles in the figure whose areas are in a 3:2 ratio. Find them, and justify your choices.

582. The transformation $\mathcal{T}(x, y) = (ax + by, cx + dy)$ sends $(13, 0)$ to $(12, 5)$ and it also sends $(0, 1)$ to $\left(-\frac{5}{13}, \frac{12}{13}\right)$. Find a, b, c, and d, then describe the nature of this transformation.

583. Let $A = (0, 5)$, $B = (-2, 1)$, $C = (6, -1)$, and $P = (12, 9)$. Let A', B', C' be the midpoints of segments PA, PB, and PC, respectively. After you make a diagram, identify the center and the magnitude of the dilation that transforms triangle ABC onto $A'B'C'$.

584. One triangle has sides that are 5 cm, 7 cm, and 8 cm long; the longest side of a similar triangle is 6 cm long. How long are the other two sides?

585. Skylar is driving along a highway that is climbing a steady 9° slope. After driving for two miles along this road, how much altitude has Skylar gained? (One mile = 5280 feet.)

586. (Continuation) How far must Skylar travel in order to gain a mile of altitude?

587. Show that $P = (3.2, 6.3)$ is not on the line $4x - y = 7$. Explain how you can tell whether P is above or below the line.

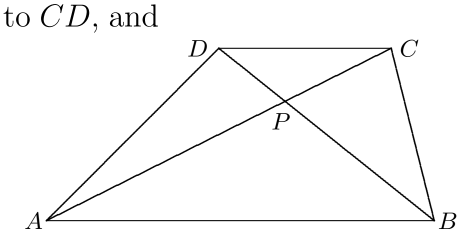
588. Explain why corresponding angles of similar polygons are necessarily the same size.

589. (Continuation) If all the angles of a triangle are equal in size to the angles of another triangle, the triangles are similar. Justify this statement. Is this the converse of the preceding?

590. Pat has \$48 and Kim has \$24, so the ratio of Pat's money to Kim's money is 2 : 1. If they both spend \$5, is the ratio still 2 : 1? Explain how they could spend their money so that the ratio of Pat's money to Kim's money remains 2 : 1.

591. Suppose that $ABCD$ is a trapezoid, with AB parallel to CD, and diagonals AC and BD intersecting at P. Explain why

(a) triangles ABC and ABD have the same area;
(b) triangles BCP and DAP have the same area;
(c) triangles ABP and CDP are similar;
(d) triangles BCP and ADP need not be similar.



592. Find the size of the acute angle formed by the intersecting lines $3x + 2y = 12$ and $x - 2y = -2$, to the nearest tenth of a degree. Do you need to find the intersection point?

593. Let $A = (0, 5, 0)$, $B = (-2, 1, 0)$, $C = (6, -1, 0)$, and $P = (2, 3, 8)$. Let A', B', C' be the midpoints of segments PA, PB, and PC, respectively. Make a diagram, and describe the relationship between triangle ABC and its image $A'B'C'$.

594. Write an equation that says that $P = (x, y)$ is 5 units from $(0, 0)$. Plot several such points. What is the configuration of all such points called? How many are lattice points?

595. The sides of a triangle are 12 cm, 35 cm, and 37 cm long.
(a) Show that this is a right triangle.
(b) Show that $\tan^{-1}$, $\sin^{-1}$, and $\cos^{-1}$ can all be used to find the size of the smallest angle of this triangle.

596. The midpoints of the sides of a quadrilateral are joined to form a new quadrilateral. For the new quadrilateral to be a rectangle, what must be true of the original quadrilateral?

597. Given the line whose equation is $y = 2x + 3$ and the points $A = (0, 0)$, $B = (1, 9)$, $C = (2, 8)$, $D = (3, 3)$, and $E = (4, 10)$, do the following:
(a) Plot the line and the points on the same axes.
(b) Let $\widehat{A}$ be the point on the line that has the same x-coordinate as A. Subtract the y-coordinate of $\widehat{A}$ from the y-coordinate of A. The result is called the *residual of* A.
(c) Calculate the other four residuals.
(d) What does a residual tell you about the relation between a point and the line?

598. The area of an equilateral triangle with m-inch sides is 8 square inches. What is the area of a regular hexagon that has m-inch sides?

599. A parallelogram has 10-inch and 18-inch sides and an area of 144 square inches.
(a) How far apart are the 18-inch sides? **(b)** How far apart are the 10-inch sides?
(c) What are the angles of the parallelogram? **(d)** How long are the diagonals?

600. Write an equation that describes all the points on the circle whose *center* is at the origin and whose *radius* is **(a)** 13; **(b)** 6; **(c)** r.

601. Campbell is about to attempt a 30-foot putt on a level surface. The hole is 4 inches in diameter. Remembering the advice of a golf pro, Campbell aims for a mark that is 6 inches from the ball and on the line from the center of the hole to the center of the ball. Campbell misses the mark by a sixteenth of an inch. Does the ball go in the hole?

602. A trapezoid has 11-inch and 25-inch parallel sides, and an area of 216 square inches.
(a) How far apart are the parallel sides?
(b) If one of the non-parallel sides is 13 inches long, how long is the other one? (N.B. There are two answers to this question. It is best to make a separate diagram for each.)

603. Graph the circle whose equation is $x^2 + y^2 = 64$. What is its radius? What do the equations $x^2 + y^2 = 1$, $x^2 + y^2 = 40$, and $x^2 + y^2 = k$ have in common? How do they differ?

604. Taylor lets out 120 meters of kite string, then wonders how high the kite has risen. Taylor is able to calculate the answer, after using a protractor to measure the 63° angle of elevation that the string makes with the ground. How high is the kite, to the nearest meter? What (unrealistic) assumptions did you make in answering this question?

605. By hand, find the sine of a 45° angle. Use a calculator to check your answer.

606. A triangle that has a 5-inch and a 6-inch side can be similar to a triangle that has a 4-inch and an 8-inch side. Find an example. Check that your example really *is* a triangle.

607. Let $A = (1,5)$, $B = (3,1)$, $C = (5,4)$, $A' = (5,9)$, $B' = (11,-3)$, and $C' = (17,6)$. Show that there is a dilation that transforms triangle ABC onto triangle $A'B'C'$. In other words, find the dilation center and the magnification factor.

608. (Continuation) Calculate the areas of triangles ABC and $A'B'C'$. Are your answers related in a predictable way?

609. The vectors $[12,0]$ and $[3,4]$ form a parallelogram. Find the lengths of its altitudes.

610. The vertices of triangle ABC are $A = (-5,-12)$, $B = (5,-12)$, and $C = (5,12)$. Confirm that the circumcenter of ABC lies at the origin. Find an equation for the circumcircle.

611. If the sides of a triangle are 13, 14, and 15 cm long, then the altitude drawn to the 14-cm side is 12 cm long. How long are the other two altitudes? Which side has the longest altitude?

612. (Continuation) How long are the altitudes of a triangle whose sides are 26, 28, and 30 cm long? What happens to the area of a triangle if its dimensions are doubled?

613. Find the length of the bisector of the smallest angle of a 3-4-5 triangle.

614. Show the lines $(x,y,z) = (5+2t, 3+2t, 1-t)$ and $(x,y,z) = (13-3r, 13+2r, 4+4r)$ are not parallel, and that they do not intersect. Such lines are called *skew*.

615. Let $A = (6,0)$, $B = (0,8)$, $C = (0,0)$. In triangle ABC, let F be the *foot* of the altitude drawn from C to side AB.
(a) Explain why the angles of triangles ABC, CBF, and ACF are the same.
(b) Find coordinates for F, and use them to calculate the exact lengths FA, FB, and FC.
(c) Compare the sides of triangle ABC with the sides of triangle ACF. What do you notice?

616. A *similarity transformation* is a geometric transformation that uniformly multiplies distances, in the following sense: For some positive number m, and any two points A and B and their respective images A' and B', the distance from A' to B' is m times the distance from A to B. You have recently shown that any dilation $\mathcal{T}(x,y) = (mx, my)$ is a similarity transformation. Is it true that the transformation $\mathcal{T}(x,y) = (3x, 2y)$ is a similarity transformation? Explain.

617. The area of the triangle determined by the vectors $\mathbf{v}$ and $\mathbf{w}$ is 5. What is the area of the triangle determined by the vectors $2\mathbf{v}$ and $3\mathbf{w}$? Justify your answer. Do not assume that $\mathbf{v}$ and $\mathbf{w}$ are perpendicular.

618. Decide whether the transformation $\mathcal{T}(x, y) = (4x - 3y, 3x + 4y)$ is a similarity transformation. If so, what is the multiplier m?

619. A rectangular sheet of paper is 21 cm wide. When it is folded in half, with the crease running parallel to the 21-cm sides, the resulting rectangle is the same shape as the unfolded sheet. Find the length of the sheet, to the nearest tenth of a cm. Note: in many places outside of the United States, such as Europe, the shape of notebook paper is determined by this similarity property.

620. How much evidence is needed to be sure that two triangles are similar?

621. A line of slope $\frac{1}{2}$ intersects a line of slope 3. Find the size of the acute angle that these lines form.

622. Use a calculator to find the sine of a 56-degree angle and the cosine of a 34-degree angle. Now find the sine of a 23-degree angle and the cosine of a 67-degree angle. The word *cosine* abbreviates "sine of the complement." Explain the terminology. The cosine function seems to be unnecessary. Explain.

623. Square $ABCD$ has 8-inch sides, M is the midpoint of BC, and N is the intersection of AM and diagonal BD. Find the lengths of NB, NM, NA, and ND.

624. Parallelogram $PQRS$ has $PQ = 8$ cm, $QR = 9$ cm, and diagonal $QS = 10$ cm. Mark F on RS, exactly 5 cm from S. Let T be the intersection of PF and QS. Find the lengths TS and TQ.

625. The parallel sides of a trapezoid are 12 inches and 18 inches long. The non-parallel sides meet when one is extended 9 inches and the other is extended 16 inches. How long are the non-parallel sides of this trapezoid?

626. The lengths of the sides of triangle ABC are often abbreviated by writing $a = BC$, $b = CA$, and $c = AB$. Notice that lower-case sides oppose upper-case vertices. Suppose now that angle BCA is right, so that $a^2 + b^2 = c^2$. Let F be the foot of the perpendicular drawn from C to the hypotenuse AB. If $a = 5$, $b = 12$ and $c = 13$, what are the lengths of FA, FB, and FC? Does $c = FA + FB$?

627. Apply the Angle-Bisector Theorem to the smallest angle of the right triangle whose sides are 1, 2, and $\sqrt{3}$. The side of length 1 is divided by the bisector into segments of what lengths? Using a calculator, check your answer for the tangent of a 15-degree angle.

628. Sketch the circle whose equation is $x^2 + y^2 = 100$. Using the same system of coordinate axes, graph the line $x + 3y = 10$, which should intersect the circle twice — at $A = (10, 0)$ and at another point B in the second quadrant. Estimate the coordinates of B. Now use algebra to find them exactly. Segment AB is called a *chord* of the circle.

629. (Continuation) What percentage of the circumference of the circle lies above the chord AB? First estimate the percentage, then find a way of calculating it precisely.

630. (Continuation) Find coordinates for a point C on the circle that makes chords AB and AC have equal length. What percentage of the circumference lies below chord AC?

631. What is the radius of the smallest circle that surrounds a 5-by-12 rectangle?

632. A triangle is defined by placing vectors $[5, 7]$ and $[-21, 15]$ tail to tail. Find its angles.

633. Without doing any calculation, what can you say about the tangent of a k-degree angle, when k is a number between 90 and 180? Explain your response, then check with a calculator.

634. What is the radius of the largest circle that you can draw on graph paper that encloses
(a) no lattice points?
(b) exactly one lattice point?
(c) exactly two lattice points?
(d) exactly three lattice points?

635. Measure the circumference and diameter of one of the 10 common circular objects supplied by your teacher. Together with your classmates, make a chart of the 10 ordered pairs (diameter, circumference).
(a) On a sheet of graph paper, make a *scatterplot* of the set of ordered pairs.
(b) Use a ruler to draw a line that fits the trend of the data. Find an equation for your line. Compare your linear equation with the results of your classmates.
(c) Give an interpretation (with units) for the slope and intercept of your line. Are there theoretical values for each of these parameters? Explain.

636. (Continuation) Not all the points in the scatterplot lie on the line you have drawn, but in some sense you have fit the line to the data. For each data point, the difference between the measured value of the circumference and the predicted value on the line at the same diameter is known as a residual. A point above the line will have a positive residual, and a point below the line will have a negative residual. Calculate the residuals for this data, and make a plot of the residuals versus the diameter for each point.

637. (Continuation) A standard method for obtaining a line of best fit for a data set is called *linear regression*, a technique that yields the line that minimizes the sum of the squares of the residuals. (This line is alternately called the *least squares line.*) Use technology to obtain an equation for the linear regression line for the diameter versus circumference data. How does this compare with the line you drew previously?

638. Let λ be the line $y = 1$ and F be the point $(-1, 2)$. Verify that the point $(2, 6)$ is equidistant from λ and F. Sketch the configuration of *all* points P that are equidistant from F and λ. Recall that this curve is called a *parabola*. Point F is called its *focus*, and line λ is called its *directrix*. Find an equation that says that $P = (x, y)$ is on the parabola.

639. (Continuation) Let $N = (2, 1)$, and find an equation for the perpendicular bisector of FN. As a check, verify that $P = (2, 6)$ is on this line. (Why could this have been predicted?) Explain why this line intersects the parabola only at P.

640. The line $y = x + 2$ intersects the circle $x^2 + y^2 = 10$ in two points. Call the third-quadrant point R and the first-quadrant point E, and find their coordinates. Let D be the point where the line through R and the center of the circle intersects the circle again. The chord DR is an example of a *diameter*. Show that triangle RED is a right triangle.

641. (Continuation) The portion of the circle that lies above chord RE is called an *arc*. Find a way of calculating and describing its *size*. The portion of the circle that lies below line RE is also an arc. The first arc is called a *minor* arc because it is less than half the circle, and the second arc is called a *major* arc because it is more than half the circle. It is straightforward to find the size of major arc RE once you know the size of minor arc RE. Explain how to do it.

642. Calculate the residual of $P = (1.2, 2.4)$ with respect to the line $3x + 4y = 12$.

643. Transformation $\mathcal{T}$ is defined by $\mathcal{T}(x, y) = (-5, 1) + 3[x + 5, y - 1]$. An equivalent definition is $\mathcal{T}(x, y) = (3x + 10, 3y - 2)$. Use the first definition to help you explain what kind of transformation $\mathcal{T}$ is.

644. Show that the area of a square is half the product of its diagonals. Then consider the possibility that there might be other quadrilaterals with the same property.

645. To the nearest tenth of a degree, calculate the angles of the triangle with vertices $(0, 0)$, $(6, 3)$, and $(1, 8)$.

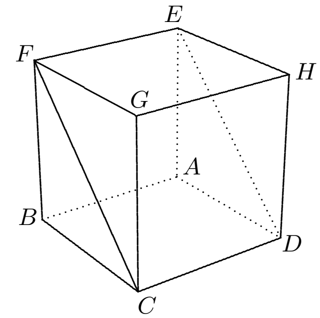
646. In a right triangle, the 58-cm hypotenuse makes a 51-degree angle with one of the legs. To the nearest tenth of a cm, how long is that leg? Once you have the answer, find two ways to calculate the length of the other leg. They should both give the same answer.

647. The sides of a square are parallel to the coordinate axes. Its vertices lie on the circle of radius 5 whose center is at the origin. Find coordinates for the four vertices of this square.

648. Find the lengths of both altitudes in the parallelogram determined by $[2, 3]$ and $[-5, 7]$.

649. Let $A = (0, 1)$, $B = (7, 0)$, $C = (3, 7)$, and $D = (0, 6)$. Find the areas of triangles ABC and ADC, which share side AC. Calculate the ratio of areas $ABC : ADC$. If you were to calculate the distances from B and D to the line AC, how would they compare? Explain your reasoning, or else calculate the two distances to confirm your prediction.

650. Draw a circle and label one of its diameters AB. Choose any other point on the circle and call it C. What can you say about the size of angle ACB? Does it depend on which C you chose? Justify your response.



651. The figure at right shows a cube $ABCDEFGH$. Square $ABCD$ and rectangle $EFCD$ form an angle that is called *dihedral* because it is formed by two intersecting planes. The line of intersection here is CD. Calculate the size of this angle.

652. *The reflection property of parabolas.* Consider the parabola whose focus is $F = (1, 4)$ and whose directrix is the line $x = -3$.
(a) Sketch the parabola, and make calculations that confirm that $P = (7, 12)$ is on it.
(b) Find the slope of the line μ through P that is *tangent* to the parabola.
(c) Calculate the size of the angle that μ makes with the line $y = 12$.
(d) Calculate the size of the angle that μ makes with segment FP. How does this support the title of the problem?

653. Draw a large triangle ABC, and mark D on segment AC so that the ratio $AD:DC$ is equal to 3:4. Mark any point P on segment BD.
(a) Find the ratio of the area of triangle BAD to the area of triangle BCD.
(b) Find the ratio of the area of triangle PAD to the area of triangle PCD.
(c) Find the ratio of the area of triangle BAP to the area of triangle BCP.

654. Suppose that triangle ABC has a 30-degree angle at A and a 60-degree angle at B. Let O be the midpoint of AB. Draw the circle centered at O that goes through A. Explain why this circle also goes through B and C. Angle BOC is called a *central angle* of the circle because its vertex is at the center. The minor arc BC is called a *60-degree arc* because it *subtends* a 60-degree angle at the center. What is the *angular size* of minor arc AC? of *major* arc AC? How does the actual length of minor arc AC compare to the length of minor arc BC?

655. A triangle has two k-inch sides that make a 36-degree angle, and the third side is one inch long. Draw the bisector of one of the other angles. How long is it? There are several ways to calculate the number k. Apply at least two of them.

656. Let $A = (0, 0)$, $B = (12, 0)$, $C = (8, 6)$, and $D = (2, 6)$. The diagonals AC and BD of trapezoid $ABCD$ intersect at P. Explain why triangle ABP is similar to triangle CDP. What is the ratio of similarity? Which side of triangle CDP corresponds to side AP in triangle ABP? Why is it inaccurate to write $\triangle ABP \sim \triangle DCP$? Without finding the coordinates of P, show how you can find the lengths AP and PC.

657. (Continuation) Find the ratio of the areas of triangles
(a) ADP to CDP; **(b)** ADP to ABP; **(c)** CDP to ABP.

658. Consider the points $A = (-0.5, -8)$, $B = (0.5, -5)$, and $C = (3, 4.5)$. Calculate the residual for each of these points with respect to the line $4x - y = 7$.

659. A projector is directed towards a screen, upon which it projects a rectangular image. The following data set gives the horizontal distance of the projector lens from the screen and the resulting length of the diagonal of the rectangular image.
(a) Obtain a scatterplot of the set of ordered pairs (distance, diagonal).
(b) Find the best-fit line for the scatterplot using the least squares criterion.
(c) Give an interpretation, if appropriate, for the slope and intercept of the least squares line. Be sure to include units.

Distance from screen (cm)	236	289	336	393	438	470	506	540
Length of diagonal (cm)	103	129	152	175	195	208	226	238

660. Draw an accurate version of a regular pentagon. Be prepared to report on the method you used to draw this figure. Measure the length of a diagonal and the length of a side. Then divide the diagonal length by the side length. When you and your classmates compare answers, on which of the preceding numbers should you agree — the lengths or the ratio?

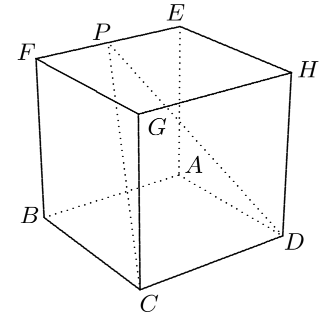
661. (Continuation) Calculate the ratio of the diagonal length to the side length in any regular pentagon. One way to do it is to use trigonometry.

662. (Continuation) Label your pentagon $ABCDE$. Draw its diagonals. They intersect to form a smaller pentagon $A'B'C'D'E'$ that lies inside $ABCDE$.
(a) Explain why $A'B'C'D'E'$ is regular, and why it is similar to $ABCDE$.
(b) Measure the length $A'B'$, and divide it by AB. Then use trigonometry to find an exact value for $A'B' : AB$, which is called the *ratio of similarity*.
(c) Consider the ways of assigning the labels A', B', C', D', and E' to the vertices of the small pentagon. How many ways are there? Is there one that stands out from the rest?

663. (Continuation) It should be possible to *circumscribe* a circle around your pentagon $ABCDE$, meaning that a circle can be drawn that goes through all five of its vertices. Find the center of this circle, and describe your method. Then measure the radius of the circle, and express your answer as a multiple of the length AB. Which of these numbers will be more useful to bring to class — the radius or the ratio?

664. If two chords of a circle have the same length, then their minor arcs have the same length too. True or false? Explain. What about the converse statement? Is it true? Why?

665. The figure at right shows a cube $ABCDEFGH$, where P is the midpoint of FE.
(a) Calculate the size of the dihedral angle formed by square $ABCD$ and triangle PCD.
(b) What happens to the angle if P is a different point on FE?



666. Draw a line λ in your notebook, and mark a point F approximately an inch away from λ. Sketch the parabola that has λ as its directrix and F as its focus. Locate the point V on the parabola, called the *vertex*, which is closest to the focus. Draw the line through F that is perpendicular to λ. How is this line related to V and to the parabola?

667. Suppose that MP is a diameter of a circle centered at O, and Q is any other point on the circle. Draw the line through O that is parallel to MQ, and let R be the point where it meets minor arc PQ. Prove that R is the midpoint of minor arc PQ.

668. Line μ (*Greek* "mu") intersects segment AB at D, forming a 57-degree angle. Suppose that $AD:DB = 2:3$ is known. What can you say about the distances from A to μ and from B to μ? If $2:3$ is replaced by another ratio $m:n$, how is your answer affected?

669. The circle $x^2 + y^2 = 25$ goes through $A = (5, 0)$ and $B = (3, 4)$. To the nearest tenth of a degree, find the size of the minor arc AB.

670. An equilateral triangle is *inscribed* in the circle of radius 1 centered at the origin (the *unit circle*). If one of the vertices is $(1, 0)$, what are the coordinates of the other two? The three points divide the circle into three arcs; what are the angular sizes of these arcs?

671. On a circle whose center is O, mark points P and A so that minor arc PA is a 46-degree arc. What does this tell you about angle POA? Extend PO to meet the circle again at T. What is the size of angle PTA? This angle is *inscribed* in the circle, because all three points are on the circle. The arc PA is *intercepted* by the angle PTA.

672. (Continuation) If minor arc PA is a k-degree arc, what is the size of angle PTA?

673. The area of triangle ABC is 231 square inches, and point P is marked on side AB so that $AP:PB = 3:4$. What are the areas of triangles APC and BPC?

674. Show that the medians of any triangle divide the triangle into six smaller triangles of equal area. Are any of the small triangles necessarily congruent to each other?

675. Triangle ABC has a 53-degree angle at A, and its circumcenter is at K. Draw a good picture of this triangle, and measure the size of angle BKC. Be prepared to describe the process you used to find K. Measure the angles B and AKC of your triangle. Measure angles C and AKB. Make a conjecture about arcs intercepted by inscribed angles. Justify your assertion.

676. The area of a trapezoidal cornfield $IOWA$ is 18000 sq m. The 100-meter side IO is parallel to the 150-meter side WA. This field is divided into four sections by diagonal roads IW and OA. Find the areas of the triangular sections.

677. In triangle ABC, it is given that angle BCA is right. Let $a = BC$, $b = CA$, and $c = AB$. Using a, b, and c, express the sine, cosine, and tangent ratios of acute angles A and B.

678. The sine of a 38-degree angle is some number r. Without using a calculator, you should be able to identify the angle size whose cosine is the same number r.

679. Draw non-parallel vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{u}+\mathbf{v}$ emanating from a common point. In order that $\mathbf{u}+\mathbf{v}$ bisect the angle formed by $\mathbf{u}$ and $\mathbf{v}$, what must be true of $\mathbf{u}$ and $\mathbf{v}$?

680. If P and Q are points on a circle, then the center of the circle must be on the perpendicular bisector of chord PQ. Explain. Which point on the chord is closest to the center? Why?

681. Given that triangle ABC is similar to triangle PQR, write the three-term proportion that describes how the six sides of these figures are related.

682. Draw a circle with a 2-inch radius, mark four points randomly (not evenly spaced) on it, and label them consecutively G, E, O, and M. Measure angles GEO and GMO. Could you have predicted the result? Name another pair of angles that would have produced the same result.

683. A circular park 80 meters in diameter has a straight path cutting across it. It is 24 meters from the center of the park to the closest point on this path. How long is the path?

684. Show that the lines $y=2x-5$ and $-2x+11y=25$ create chords of equal length when they intersect the circle $x^2+y^2=25$. Make a large diagram, and measure the inscribed angle formed by these chords. Describe two ways of calculating its size to the nearest 0.1 degree. What is the angular size of the arc that is intercepted by this inscribed angle?

685. A triangle has a 3-inch side, a 4-inch side, and a 5-inch side. The altitude drawn to the 5-inch side cuts this side into segments of what lengths?

686. The parallel sides of a trapezoid are 8 inches and 12 inches long, while one of the non-parallel sides is 6 inches long. How far must this side be extended to meet the extension of the opposite side? What are the possible lengths for the opposite side?

687. The midline of a trapezoid is *not* concurrent with the diagonals. Explain why.

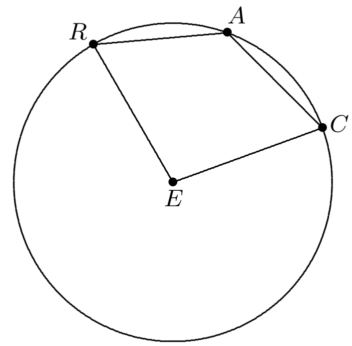
688. A chord 6 cm long is 2 cm from the center of a circle. How long is a chord that is 1 cm from the center of the same circle?

689. By using the triangle whose sides have lengths 1, $\sqrt{3}$, and 2, express the sine, cosine, and tangent of a 30-degree angle as ratios of these lengths. You can use a calculator to check your answers, of course.

690. Triangle ABC is inscribed in a circle. Given that AB is a 40-degree arc and ABC is a 50-degree angle, find the sizes of the other arcs and angles in the figure.

691. Suppose that chords AB and BC have the same lengths as chords PQ and QR, respectively, with all six points belonging to the same circle (they are *concyclic*). Is this enough information to conclude that chords AC and PR have the same length? Explain.

692. The figure at right shows points C, A, and R marked on a circle centered at E, so that chords CA and AR have the same length, and so that major arc CR is a 260-degree arc. Find the angles of quadrilateral $CARE$. What is special about the sizes of angles CAR and ACE?



693. The sides of a triangle are found to be 10 cm, 14 cm, and 16 cm long, while the sides of another triangle are found to be 15 in, 21 in, and 24 in long. What can you say about the angles of these triangles? Show how to calculate their sizes.

694. The points A, P, Q, and B appear in this order on a line, so that $AP:PQ = 2:3$ and $PQ:QB = 5:8$. Find whole numbers that are proportional to $AP:PQ:QB$.

695. A trapezoid has two 65-degree angles, and also 8-inch and 12-inch parallel sides. How long are the non-parallel sides? What is the area enclosed by this figure?

696. The data shown was generated by suspending weights, whose mass is measured in kg, from a rubber band, which stretches by the amount shown, measured in meters. Use technology to find the least squares line for this data set and interpret the results. What is the meaning of the slope in the context of this problem? How much stretch does your model predict for a 4.20 kg weight?

mass	*stretch*
0.49	0.007
0.68	0.013
1.19	0.023
1.96	0.053
2.94	0.110
3.43	0.144
4.41	0.195
5.39	0.273

697. (Continuation) The previous prediction is an example of *interpolation*, which is a prediction within the interval of values found in the data. *Extrapolation* is a prediction outside the range of the data. Is there a reason you might want to be cautious about extrapolating?

698. The dimensions of rectangle $ABCD$ are $AB = 12$ and $BC = 16$. Point P is marked on side BC, so that $BP = 5$, and the intersection of AP and BD is called T. Find the lengths of the four segments TA, TP, TB, and TD.

699. Two circles of radius 10 cm are drawn so that their centers are 12 cm apart. The two points of intersection determine a *common chord*. Find the length of this chord.

700. What is the sine of the angle whose tangent is 2? First find an answer by hand (draw a picture), then use a calculator to check.

701. Consider the line $y = 1.8x + 0.7$.
(a) Find a point whose residual with respect to this line is -1.
(b) Describe the configuration of points whose residuals are -1 with respect to this line.

702. The *median* of a set of numbers is the middle number, once the numbers have been arranged in order from least to greatest. If there are two middle numbers, then the median is half their sum. Find the median of **(a)** 5, 8, 3, 9, 5, 6, 8; **(b)** 4, 10, 8, 7.

703. True or false? The midline of a trapezoid divides the figure into two trapezoids, each similar to the original. Explain.

704. Hilary and Dale leave camp and go for a long hike. After going 7 km due east, they turn and go another 8 km in the direction 60 degrees north of east. They plan to return along a straight path. How far from camp are they at this point? Use an angle to describe the direction that Hilary and Dale should follow to reach their camp.

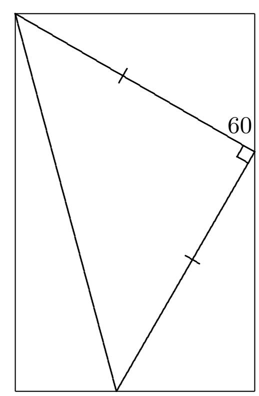
705. A right triangle has 6-inch, 8-inch, and 10-inch sides. A square can be inscribed in this triangle, with one vertex on each leg and two vertices on the hypotenuse. How long are the sides of the square?

706. Find a triangle two of whose angles have sizes $\tan^{-1}(1.5)$ and $\tan^{-1}(3)$. Answer this question either by giving coordinates for the three vertices, or by giving the lengths of the three sides. To the nearest 0.1 degree, find the size of the third angle in your triangle.

707. The area of an equilateral triangle is $100\sqrt{3}$ square inches. How long are its sides?

708. Points P, E, and A are marked on a circle whose center is R. In quadrilateral $PEAR$, angles A and E are found to be 54° and 113°, respectively. What are the other two angles?

709. The diagram shows a rectangle that has been formed by bordering an isosceles right triangle with three other right triangles, one of which has a 60-degree angle as shown. Find the sizes of the other angles in the figure. By assigning lengths to all the segments, find values for the sine, cosine, and tangent of a 75-degree angle, *without using a calculator's trigonometric functions* (except to check your formulas).



710. The points $A = (0, 13)$ and $B = (12, 5)$ lie on a circle whose center is at the origin. Write an equation for the perpendicular bisector of segment AB. Notice that this bisector goes through the origin; why was this expected?

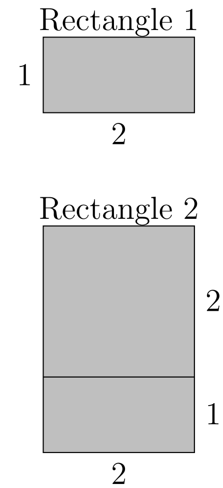
711. (Continuation) Find center and radius for another circle to which A and B both belong, and write an equation for it. How small can such a circle be? How large? What can be said about the centers of all such circles?

712. The areas of two similar triangles are 24 square cm and 54 square cm. The smaller triangle has a 6-cm side. How long is the corresponding side of the larger triangle?

713. When two circles have a common chord, their centers and the endpoints of the chord form a quadrilateral. What kind of quadrilateral is it? What special property do its diagonals have?

714. The area of triangle ABC is 75 square cm. Medians AN and CM intersect at G. What is the area of quadrilateral $GMBN$?

715. Construct a sequence of rectangles using the following procedure: The first rectangle measures 2×1 (2 units by 1 unit). Form the second rectangle by fitting a square to the longer side. (This yields a rectangle that measures 3×2.) Continue to add squares to the longer side of each successive rectangle. (The third rectangle measures 5×3.)



(a) Make a table of values for the width (length of the shorter side) versus length for the first five rectangles.
(b) Make a scatterplot showing width versus length and find the least squares line for the data.
(c) Examine what happens to the slope and intercept of the least squares line as you add the point for the sixth rectangle, then the seventh.
(d) The sequence of values for the widths of the rectangles is an example of a Fibonacci sequence. What does the slope of the least squares line represent?

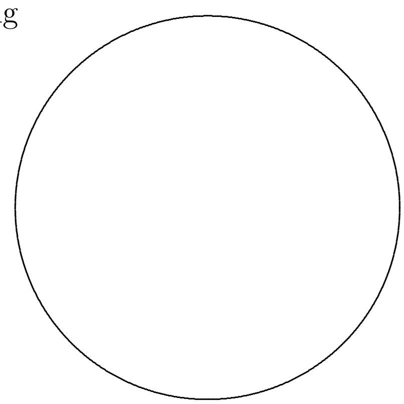
716. Given that θ (*Greek* "theta") stands for the degree size of an acute angle, fill in the blank space between the parentheses to create a true statement: $\sin\theta = \cos(\qquad)$.

717. If corresponding sides of two similar triangles are in a $3:5$ ratio, then what is the ratio of the areas of these triangles?

718. Let $P = (-25, 0)$, $Q = (25, 0)$, and $R = (-24, 7)$.
(a) Find an equation for the circle that goes through P, Q, and R.
(b) Find at least two ways of showing that angle PRQ is right.
(c) Find coordinates for three other points R that would have made angle PRQ right.

719. Show that $(-2, 10)$, $(1, 11)$, $(6, 10)$, and $(9, 7)$ are concyclic.

720. Explain how to find the center of the circle shown, using only a pencil and a rectangular sheet of paper.



721. Trapezoid $ABCD$ has parallel sides AB and CD, of lengths 8 and 16, respectively. Diagonals AC and BD intersect at E, and the length of AC is 15. Find the lengths of AE and EC.

722. (Continuation) Through E draw the line parallel to sides AB and CD, and let P and Q be its intersections with DA and BC, respectively. Find the length of PQ.

723. Let $A = (0, 0)$, $B = (4, 0)$, and $C = (4, 3)$. Mark point D so that ACD is a right angle and DAC is a 45-degree angle. Find coordinates for D. Find the tangent of angle DAB.

724. Find a point on the line $y = x$ that lies on the parabola whose focus is $(0, 2)$ and directrix is the x-axis. Describe the relationship between the line $y = x$ and the parabola.

725. Two circles have a 24-cm common chord, their centers are 14 cm apart, and the radius of one of the circles is 13 cm. Make an accurate drawing, and find the radius for the second circle in your diagram. There are two solutions; find both.

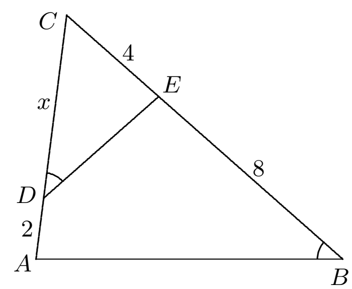
726. *SAS Similarity.* Use your protractor to carefully draw a triangle that has a 5-cm side, a 9-cm side, and whose *included angle* is 40 degrees. Construct a second triangle that has a 10-cm side, an 18-cm side, and whose included angle is also 40 degrees. Measure the remaining parts of these triangles. Could you have anticipated the results? Explain.

727. Find the perimeter of a regular 36-sided polygon *inscribed* in a circle of radius 20 cm.

728. Find the area of a regular 36-sided polygon inscribed in a circle of radius 20 cm.

729. The position of a starship is given by the equation $P_t = (18 + 3t, 24 + 4t, 110 - 5t)$. For what values of t is the starship within 100 units of a space station placed at the origin?

730. Point $P = (x, y)$ is 6 units from $A = (0, 0)$ and 9 units from $B = (9, 0)$. Find x and y.



731. Refer to the figure, in which angles ABE and CDE are equal in size, and various segments have been marked with their lengths. Find x.

732. Let $A = (0, 0)$, $B = (7, 0)$, and $C = (7, 5)$. Point D is located so that angle ACD is a right angle and the tangent of angle DAC is 5/7. Find coordinates for D.

733. A kite has an 8-inch side and a 15-inch side, which form a right angle. Find the length of the diagonals of the kite.

734. Mark points A and C on a clean sheet of paper, then spend a minute or so drawing rectangles $ABCD$. What do you notice about the configuration of points B and D?

735. What is the radius of the circumscribed circle for a triangle whose sides are 15, 15, and 24 cm long? What is the radius of the smallest circle that contains this triangle?

736. Find an equation for the circle of radius 5 whose center is at $(3, -1)$.

737. Draw a *cyclic* quadrilateral $SPAM$ in which the size of angle SPA is 110 degrees. What is the size of angle AMS? Would your answer change if M were replaced by a different point on major arc SA?

738. Let $A'B'C'$ be the midpoint triangle of triangle ABC. In other words, A', B', and C' are the midpoints of segments BC, CA, and AB, respectively. Show that triangles $A'B'C'$ and ABC have the same centroid.

739. Does $(1, 11)$ lie on the parabola defined by the focus $(0, 4)$ and the directrix $y = x$? Justify your answer.

740. Out for a walkabout in Chicago, Crocodile Dundee measures the angle of elevation to the top of the distant Willis Tower at various points along a direct path towards the building. Recall, the Willis tower was calculated to be 439 meters tall in #537. The following data set pairs the angle of elevation in degrees with the distance walked in kilometers.
(a) Obtain the least squares line for this data set, and graph the line with a scatterplot of the data. Does the line seem to fit the data?
(b) How far away was Dundee when he began walking?
(c) What will happen to the angle of elevation as Dundee gets close to the building? Does the linear model accurately predict those extrapolated values?

Distance walked (km)	0.0	0.4	0.7	1.0	1.2	1.4
Angle of elevation (degrees)	3.6	3.8	4.0	4.2	4.3	4.5

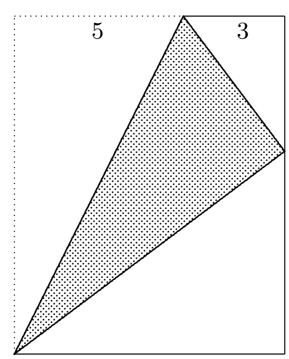
741. The area of a trapezoid is 3440 square inches, and the lengths of its parallel sides are in a 3:5 ratio. A diagonal divides the trapezoid into two triangles. What are their areas?

742. Let $WISH$ be a cyclic quadrilateral, and K be the intersection of its diagonals WS and HI. Given that arc WI is 100 degrees and arc SH is 80 degrees, find the sizes of as many angles in the figure as you can.

743. Let $A = (0, 0)$ and $B = (0, 8)$. Plot several points P that make APB a 30-degree angle. Use a protractor, and be prepared to report coordinates for your points. Formulate a conjecture about the configuration of all such points.

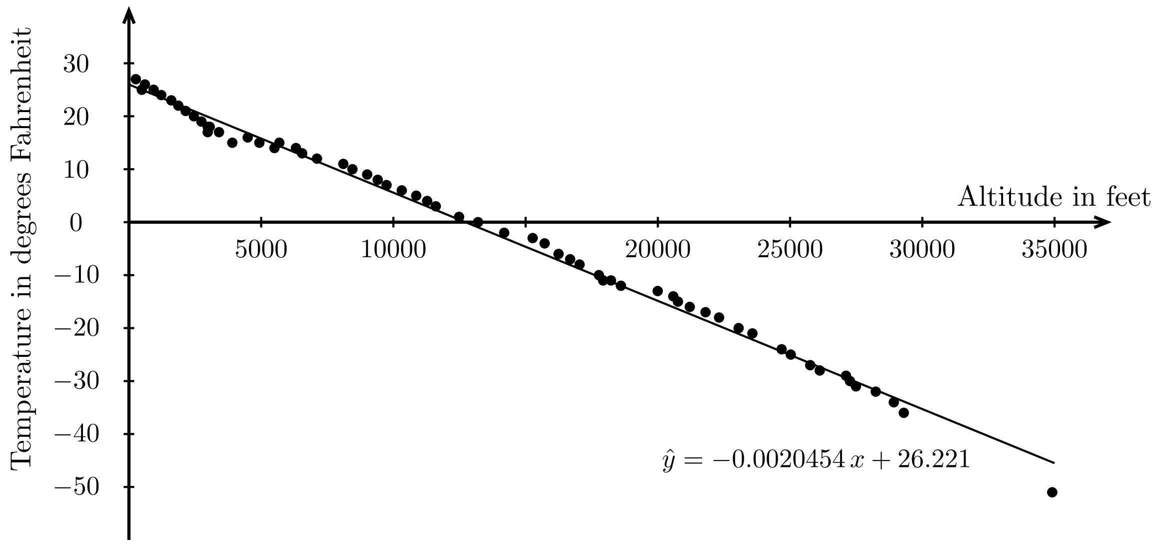
744. Triangle ABC has P on AC, Q on AB, and angle APQ equal to angle B. The lengths $AP = 3$, $AQ = 4$, and $PC = 5$ are given. Find the length of AB.

745. The figure at right shows a rectangular sheet of paper that has been creased so that one of its corners matches a point on a non-adjacent edge. Given the dimensions marked on the figure, you are to determine the length of the crease.



746. Draw the line $y = 2x - 5$ and the circle $x^2 + y^2 = 5$. Use algebra to show that these graphs touch at only one point. Find the slope of the segment that joins this point to the center of the circle, and compare your answer with the slope of the line $y = 2x - 5$. It is customary to say that a line and a circle are *tangent* if they have exactly one point in common.

747. Recently one of our math teachers was flying into Boston's Logan Airport and decided to use the flight data on the flight monitor to record the outside temperature during the descent. The collected data is shown in the graph along with the equation for the least squares line.



(a) This data exhibits a *negative association* between temperature and altitude. What does this terminology imply about the relationship between temperature and altitude?
(b) State the slope and intercept from the regression equation. What is the meaning of each number in the context of this data? Include units in your explanation.
(c) Predict the temperature at an altitude of 30000 feet.
(d) Is it appropriate to extrapolate temperatures for altitudes outside the range of altitudes shown in the data set? Why or why not?

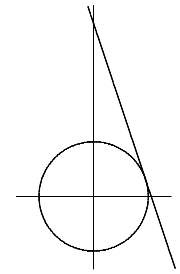
748. The length of segment AB is 20 cm. Find the distance from C to AB, given that C is a point on the circle that has AB as a diameter, and that
(a) $AC = CB$; **(b)** $AC = 10$ cm; **(c)** $AC = 12$ cm.

749. Quadrilateral $BAKE$ is cyclic. Extend BA to a point T outside the circle, thus producing the exterior angle KAT. Why do angles KAT and KEB have the same size?

750. A kite has a 5-inch side and a 7-inch side. One of the diagonals is bisected by the other. The bisecting diagonal has length 8 inches. Find the length of the bisected diagonal.

751. Drawn in a circle whose radius is 12 cm, chord AB is 16 cm long. Calculate the angular size of minor arc AB.

752. The graph of $x^2 - 6x + 9 + y^2 + 2y + 1 = 25$ is a circle. Where is the center of the circle? What is the radius of the circle?



753. Show that the line $y = 10 - 3x$ is tangent to the circle $x^2 + y^2 = 10$. Find an equation for the line perpendicular to the tangent line at the point of tangency. Show that this line goes through the center of the circle.

754. Let $K = (0, 0)$, $L = (12, 0)$, and $M = (0, 9)$. Find equations for the three lines that bisect the angles of triangle KLM. Show that the lines are concurrent at a point C, the *incenter* of KLM. Why is C called this?

755. The altitude drawn to the hypotenuse of a right triangle divides the hypotenuse into two segments, whose lengths are 8 inches and 18 inches. How long is the altitude?

756. Write an equation for the circle that is centered at $(-4, 5)$ and tangent to the x-axis.

757. Verify that the point $A = \left(8, \frac{25}{3}\right)$ lies on the parabola whose focus is $(0, 6)$ and whose directrix is the x-axis. Find an equation for the line that is tangent to the parabola at A.

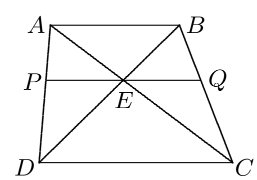
758. Let $A = (1, 3)$, $B = (6, 0)$, and $C = (9, 9)$. Find the size of angle BAC. There is more than one way to do it.

759. For (a), find center and radius. For (b), explain why it has the same graph as (a).
(a) $(x - 5)^2 + (y + 3)^2 = 49$ **(b)** $x^2 - 10x + y^2 + 6y = 15$

760. For each of the following, fill in the blank to create a perfect-square trinomial:
(a) $x^2 - 6x + \underline{\qquad}$ **(b)** $y^2 + 7y + \underline{\qquad}$ **(c)** $x^2 - 0.4x + \underline{\qquad}$ **(d)** $y^2 - \underline{\qquad}\, y + 42.25$

761. Find the center and the radius of the following circles:
(a) $x^2 + y^2 - 6x + y = 3$ **(b)** $x^2 + y^2 + 8x = 0$ **(c)** $x^2 + y^2 + 2x - 8y = -8$

762. Let $K = (5, 12)$, $L = (14, 0)$, and $M = (0, 0)$. The line $x + 2y = 14$ bisects angle MLK. Find equations for the bisectors of angles KML and MKL. Is the slope of segment MK twice the slope of the bisector through M? Should it have been? Show that the three lines concur at a point C. Does C have any special significance?



763. Trapezoid $ABCD$ has parallel sides AB and CD, of lengths 12 and 18, respectively. Diagonals AC and BD intersect at E. Draw the line through E that is parallel to AB and CD, and let P and Q be its intersections with DA and BC, respectively. Find PQ.

764. The point $P = (4, 3)$ lies on the circle $x^2 + y^2 = 25$. Write an equation in standard form for the line that is tangent to the circle at P. This line meets the x-axis at a point Q. Find an equation for the other line through Q that is tangent to the circle, and identify its point of tangency. Hmm...

765. Let $P = (4, 4, 7)$, $A = (0, 0, 0)$, $B = (8, 0, 0)$, $C = (8, 8, 0)$, and $D = (0, 8, 0)$. These points are the vertices of a *regular square pyramid*. Sketch it. To the nearest tenth of a degree, find the size of the dihedral angle formed by *lateral face* PCD and *base* $ABCD$.

766. (Continuation) Find the size of the angle formed by the edge PB and the base plane $ABCD$. First you will have to decide what this means.

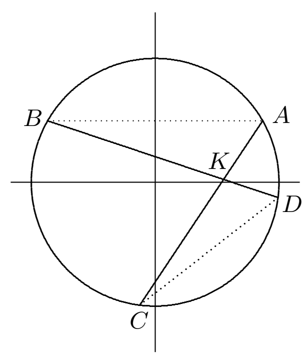
767. (Continuation) Let $Q = (5, 5, 7)$. The five points $QABCD$ are the vertices of a square pyramid. Explain why the pyramid is not regular. To the nearest tenth of a degree, find the size of the dihedral angle formed by the lateral face QCD and the base $ABCD$.

768. How long is the common chord of the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 4x$?

769. Draw the circles $x^2 + y^2 = 5$ and $(x - 2)^2 + (y - 6)^2 = 25$ on the same coordinate-axis system. Subtract one equation from the other, and simplify the result. This should produce a linear equation; graph it. Is there anything special about this line? Make a conjecture about what happens when one circle equation is subtracted from another.

770. Prove that the arcs between any two parallel chords in a circle must be the same size.

771. *Crossed Chords.* Verify that $A = (7, 4)$, $B = (-7, 4)$, $C = (-1, -8)$, and $D = (8, -1)$ all lie on a circle centered at the origin. Let K be the intersection of chords AC and BD. Prove that triangles KAB and KDC are similar and find the ratio of similarity. Then, show that $KA \cdot KC = KB \cdot KD$.



772. (Continuation) Explain why triangle KAD is similar to triangle KBC. What is the ratio of similarity? Is it the same ratio as for the other pair of similar triangles?

773. *Two-Tangent Theorem.* From any point P outside a given circle, there are two lines through P that are tangent to the circle. Explain why the distance from P to one of the points of tangency is the same as the distance from P to the other point of tangency. What special quadrilateral is formed by the center of the circle, the points of tangency, and P?

774. A 72-degree arc AB is drawn in a circle of radius 8 cm. How long is chord AB?

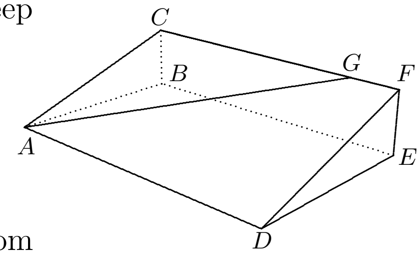
775. Find the perimeter of a regular 360-sided polygon that is inscribed in a circle of radius 5 inches. What would the perimeter be if the regular polygon had n-sides instead? If someone did not remember the formula for the circumference of a circle, show how they could use an n-sided regular polygon to estimate the circumference of a circle with a 5-inch radius?

776. The line drawn tangent to the circle $x^2 + y^2 = 169$ at $(12, 5)$ meets the y-axis where?

777. The segments GA and GB are tangent to a circle at A and B, and AGB is a 60-degree angle. Given that $GA = 12$ cm, find the distance from G to the nearest point on the circle.

778. Through the point $(13, 0)$, there are two lines that can be drawn tangent to the circle $x^2 + y^2 = 25$. Find an equation for one of them. To begin your solution, you could find the common *length* of the tangent segments.

779. Peyton's workout today is to run repeatedly up a steep grassy slope, represented by $ADFC$ in the diagram. The workout loop is $AGCA$, in which AG requires exertion and GCA is for recovery. Point G was chosen on the ridge CF to make the slope of the climb equal 1/5. Given that $ADEB$ and $BEFC$ are rectangles, ABC is a right angle, $AD = 240$, $DE = 150$, and $EF = 50$, find the distance from point G to point C.



780. (Continuation) Peyton's next workout loop is $AHCA$, where H is a point on the path AG, chosen to make the slope of HC equal 1/5. Find the ratio AH/AG, and explain your choice.

781. A circle goes through the points A, B, C, and D consecutively. The chords AC and BD intersect at P. Given that $AP = 6$, $BP = 8$, and $CP = 3$, how long is DP?

782. Write an equation that says that $P = (x, y)$ is on the parabola whose focus is $(2, 1)$ and whose directrix is the line $y = -1$.

783. *Crossed Chords Revisited.* Suppose that A, B, D, and C lie (in that order) on a circle, and that chords AC and BD intersect, when extended, at a point P that is *outside* the circle. Explain why $PA \cdot PC = PB \cdot PD$.

784. When a regular polygon is inscribed in a circle, the circle is divided into arcs of equal size. The angular size of these arcs is simply related to the size of the interior angles of the polygon. Describe the relationship.

785. A piece of a broken circular gear is brought into a metal shop so that a replacement can be built. A ruler is placed across two points on the rim, and the length of the chord is found to be 14 inches. The distance from the midpoint of this chord to the nearest point on the rim is found to be 4 inches. Find the radius of the original gear.

786. The intersecting circles $x^2 + y^2 = 100$ and $(x - 21)^2 + y^2 = 289$ have a common chord. Determine, by hand, the length of the chord.

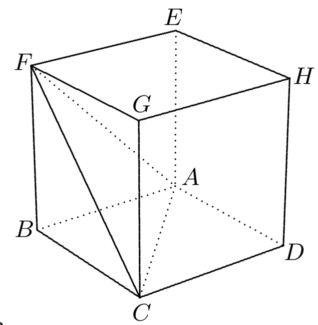
787. (Continuation) The region that is inside both circles is called a *lens.* Find the angular sizes of the two arcs that form the boundary of the lens. Does the common chord of the circles serve as a line of symmetry of the lens?

788. A triangle has two 13-cm sides and a 10-cm side. The largest circle that fits inside this triangle meets each side at a point of tangency. These points of tangency divide the sides of the triangle into segments of what lengths? What is the radius of the circle?

789. A 20-inch chord is drawn in a circle with a 12-inch radius. What is the *angular size* of the minor arc of the chord? What is the *length* of the arc, to the nearest tenth of an inch?

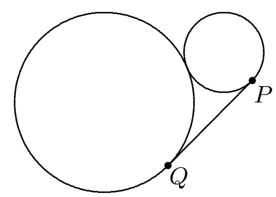
790. A triangle that has a 50-degree angle and a 60-degree angle is inscribed in a circle of radius 25 inches. The circle is divided into three arcs by the vertices of the triangle. To the nearest tenth of an inch, find the lengths of these three arcs.

791. The figure at right shows a cube $ABCDEFGH$. Triangles ABC and AFC form a dihedral angle. Notice that the line of intersection is AC. Calculate the size of this angle, to the nearest tenth of a degree.



792. What graph is traced by the parametric equation $(x, y) = (t, 4 - t^2)$? Try it by hand and with technology.

793. A circle with a 4-inch radius is centered at A, and a circle with a 9-inch radius is centered at B, where A and B are 13 inches apart. There is a segment that is tangent to the small circle at P and to the large circle at Q. It is a common external tangent of the two circles. What kind of quadrilateral is $PABQ$? What are the lengths of its sides?

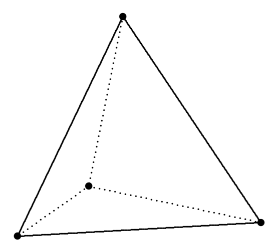


794. Segment AB, which is 25 inches long, is the diameter of a circle. Chord PQ meets AB perpendicularly at C, where $AC = 16$ in. Find the length of PQ.

795. Find the radius of the largest circle that can be drawn inside the right triangle that has 6-cm and 8-cm legs.

796. The segments GA and GB are tangent to a circle at A and B, and AGB is a 48-degree angle. Given that $GA = 12$ cm, find the distance from G to the nearest point on the circle.

797. A regular tetrahedron is a triangular pyramid, all of whose edges have the same length. If all the edges are 6-inch segments, how tall is such a pyramid, to the nearest hundredth of an inch?



798. The line $x + 2y = 5$ divides the circle $x^2 + y^2 = 25$ into two arcs. Calculate their lengths. The interior of the circle is divided into two regions by the line. Calculate their areas. Give three significant digits for your answers.

799. Within a given circle, is the length of a minor arc proportional to the length of its chord? Explain your answer.

800. As wildlife biologists monitor the health of animals, they would like measure the weight, which can be difficult to do with large animals in the wild. In the case of the Rocky Mountain elk, researchers decided to estimate weights by a much simpler method: measuring the chest girth. The following table gives the chest girth and weight of 19 selected elk. ["Estimating Elk Weight From Chest Girth" (Wildlife Society Bulletin [1996]: 58-61)]
(a) Find the equation for the least squares line.
(b) Plot the least squares line and a scatterplot of the data. Evaluate how well the line fits the data. Is the line a reasonable model for predicting weight from girth?
(c) Predict the weight of an elk that has a chest girth of 149 cm.
(d) Interpret the intercept and slope of the least squares line, if appropriate.

Girth (cm)	96	105	108	109	110	114	122	124	132
Weight (kg)	99	196	163	196	183	171	230	225	211

Girth (cm)	135	137	138	140	142	155	157	157	159	162
Weight (kg)	231	225	266	241	264	337	284	292	300	338

801. Find an equation for the circle that goes through the points $(0, 0)$, $(0, 8)$, and $(6, 12)$. Find an equation for the line that is tangent to this circle at $(6, 12)$.

802. Can a circle always be drawn through three given points? If so, describe a procedure for finding the center of the circle. If not, explain why not.

803. A dilation $\mathcal{T}$ sends $A = (2, 3)$ to $A' = (5, 4)$, and it sends $B = (3, -1)$ to $B' = (7, -4)$. Where does it send $C = (4, 1)$? Write a general formula for $\mathcal{T}(x, y)$.

804. Find an equation for the line that goes through the two intersection points of the circle $x^2 + y^2 = 25$ and the circle $(x - 8)^2 + (y - 4)^2 = 65$.

805. All triangles and rectangles have circumscribed circles. Is this true for all kites, trapezoids, and parallelograms? Which quadrilaterals have circumscribed circles? Explain.

806. Two of the tangents to a circle meet at Q, which is 25 cm from the center. The circle has a 7-cm radius. To the nearest tenth of a degree, find the angle formed at Q by the tangents.

807. To the nearest tenth of a degree, find the angle formed by placing the vectors $[4, 3]$ and $[-7, 1]$ tail to tail.

808. Four points on a circle divide it into four arcs, whose sizes are 52 degrees, 116 degrees, 100 degrees, and 92 degrees, in consecutive order. The four points determine two intersecting chords. Find the sizes of the angles formed by the intersecting chords.

809. Let $A = (3, 4)$ and $B = (-3, 4)$, which are both on the circle $x^2 + y^2 = 25$. Let λ be the line that is tangent to the circle at A. Find the angular size of minor arc AB, then find the size of the acute angle formed by λ and chord AB. Is there a predictable relation between the two numbers? Explain.

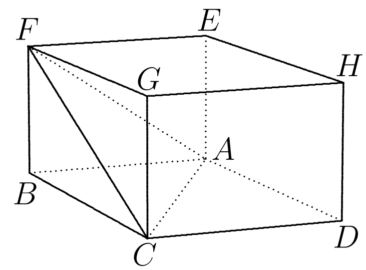
810. What is the radius of the largest circle that will fit inside a triangle that has two 15-inch sides and an 18-inch side?

811. If a line cuts a triangle into two pieces of equal area, must that line go through the centroid of the triangle? Explain your answer.

812. *An Apollonian circle.* Let $A = (-5, 0)$ and $B = (1, 0)$. Plot a few points $P = (x, y)$ for which $PA = 2PB$, including any that lie on the coordinate axes. Use the distance formula to find an equation for the configuration of all such points. Simplify your equation. Does it help you identify your graph?

813. One stick is three times as long as another. You break the longer stick at a random point. Now you have three sticks. What is the probability that they form a triangle?

814. The figure at right shows a rectangular *prism* $ABCDEFGH$, with lengths: $CD = 8$, $DH = 5$, and $EH = 6$. Triangles ABC and AFC form a dihedral angle. To the nearest tenth of a degree, calculate the size of this dihedral angle.



815. Four points on a circle divide it into four arcs, whose sizes are 52 degrees, 116 degrees, 100 degrees, and 92 degrees, in consecutive order. Use technology to create an accurate diagram. When extended, the chord that belongs to the 52-degree arc intersects the chord that belongs to the 100-degree arc, at a point P outside the circle. Find the size of angle P.

816. A chord AB in a circle is extended to a point P outside the circle, and then PT is drawn tangent to the circle at T.
(a) Show that angles TAB and PTB are the same size.
(b) Show that $PT \cdot PT = PA \cdot PB$.

817. The Geogebra simulation found at www.geogebra.org/m/xfz4dfaw allows you to guess the least squares line for a given data set. The simulation allows you to change the equation of the line by dragging one of two points on the line that is shown on the graph of the data points. It also shows visually the squares (literally) of the residuals. As you move the line around the scatterplot, what is going on visually? What is the significance of the squares? How does the graph appear when you finally arrive at the least squares line?

818. The Geogebra simulation www.geogebra.org/m/qkxmrxdu allows the user to examine the effect of a single point on the least squares line fit to a data set for the chest girth and weight of 9 elk.
(a) The slider for a moves vertically a central point in the data set. What do you notice about the least squares line as you change the location of this point?
(b) The slider for b moves vertically a point with the lowest x-value of the data set. What do you notice about the least squares line as you change the location of this point?
(c) Compare and contrast the results for parts (a) and (b).

819. In the Assembly Hall one day, Tyler spends some time trying to figure which row gives the best view of the screen. The screen is 18 feet tall, and its bottom edge is 6 feet above eye level. Tyler finds that sitting 36 feet from the plane of the screen is not satisfactory, for the screen is far away and subtends only a 24.2-degree angle. Verify this. Sitting 4 feet from the screen is just as bad, because the screen subtends the same 24.2-degree angle from this position. Verify this also.

820. (Continuation)
(a) Intrigued by this surprising coincidence, Tyler now tries a few other distances and notices that the angles subtended by the screen at both 8 feet and 18 feet from the screen are also the same, though larger than 24.2 degrees. Find this new angle.
(b) If Tyler sits 9 feet from the screen, the angle subtended by the screen will increase, and it will equal the angle subtended when sitting k feet from the screen. Find k. See if you can determine the relationship between each of these pairs of distances that subtend equal angles.
(c) Now find the optimal viewing distance, that distance that makes the angle subtended by the screen as large as possible, and find that angle.

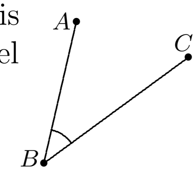
Mathematics 2 Reference

AAA similarity: Two *triangles* are sure to be similar if their angles are equal in size.

adjacent: Two vertices of a polygon that are connected by an edge. Two edges of a polygon that intersect at a vertex. Two angles of a polygon that have a common side.

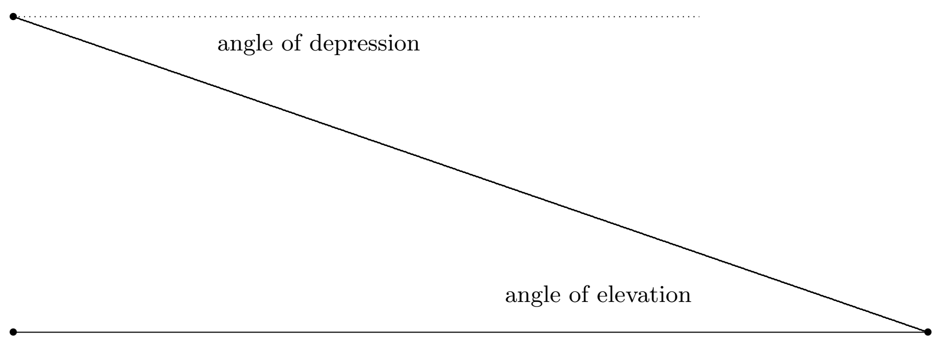
Alex in the desert: [110, 157, 331, 367, 539]

altitude: In a triangle, an altitude is a segment that joins one of the three vertices to a point on the opposite side, the intersection being perpendicular. In some triangles, it may be necessary to extend the side to meet the altitude. The *length* of this segment is also called an altitude, as is the distance that separates the parallel sides of a trapezoid. [173, 345, 402, 404]



angles can often be identified by a single letter, but sometimes three letters are necessary. The angle shown can be called B, ABC, or CBA. [7]

angle of depression: Angle formed by a horizontal ray and a line-of-sight ray that is below the horizontal. See the diagram below. [480]



angle of elevation: Angle formed by a horizontal ray and a line-of-sight ray that is above the horizontal. See the diagram above. [467]

Angle-Angle-Side (corresponding): When the parts of one triangle can be matched with the parts of another triangle, so that two pairs of corresponding angles have the same sizes, and so that one pair of corresponding sides has the same length, then the triangles are congruent. This rule of evidence is abbreviated to AAS. [120, 108]

angle bisector: Given an angle, this ray divides the angle into two equal parts. [201, 203]

Angle-Bisector Theorem: The bisector of any angle of a triangle cuts the opposite side into segments whose lengths are proportional to the adjacent sides that form the angle. [479]

Angle-Side-Angle: When the parts of one triangle can be matched with the parts of another triangle, so that two pairs of corresponding angles have the same sizes, and so that the (corresponding) shared sides have the same length, then the triangles are congruent. This rule of evidence is abbreviated to ASA. [120]

Mathematics 2 Reference

angular size of an arc: This is the size of the central angle formed by the radii that meet the endpoints of the arc. [654]

Apollonian circle: A curve consisting of those points whose distances from two fixed points are in a constant ratio. [448, 812]The Greek geometer Apollonius of Perga, who flourished about 2200 years ago, wrote many books, and gave the *parabola* its name.

arc: The portion of a circle that lies to one side of a chord is called an *arc*. [641]

arc length: The length of any arc of a circle is proportional to the size of its central angle.

areas of similar figures: If two figures are similar, then the ratio of their areas equals the *square* of the ratio of similarity. [608, 609, 712, 717]

arithmetic mean: For a set of numbers, the arithmetic mean is the sum of all the numbers in the set divided by the quantity of numbers in the set. The arithmetic mean of a and b is $\frac{a+b}{2}$. Sometimes the arithmetic mean is simply referred to as the "mean" or "average." Note that the arithmetic mean is different from the geometric mean and the harmonic mean.

bagel: [151, 469]

bisect: Divide into two pieces that are, in some sense, equal. [30, 31, 47, 183, 264]

buckyball: Named in honor of R. Buckminster Fuller, this is just another name for the *truncated icosahedron*. [195]

central angle: An angle formed by two radii of a circle. [654]

centroid: The medians of a triangle are concurrent at this point, which is the balance point (also known as the *center of gravity*) of the triangle. [216, 413, 438]

chord: A segment whose endpoints lie on a circle is called a *chord* of the circle. [628]

circle: This curve consists of all points that are at a constant distance from a *center*. The common distance is the *radius* of the circle. A segment joining the center to a point on the circle is also called a *radius*. [594, 600]

circumcenter: The perpendicular bisectors of the sides of a triangle are concurrent at this point, which is equidistant from the vertices of the triangle. [235, 236]

circumcircle: When possible, the circle that goes through all the vertices of a polygon.

collinear: Three (or more) points that all lie on a single line are *collinear*. [49, 57]

common chord: The segment that joins the points where two circles intersect. [699]

complementary: Two angles that fit together to form a right angle are called complementary. Each angle is the *complement* of the other. [26]

completing the square: Applied to an equation, this is an algebraic process that is useful for finding the center and the radius of a circle, or the vertex and focus of a parabola. [759, 760, 761]

components describe how to move from one unspecified point to another. They are obtained by *subtracting* coordinates. [67]

concentric: Two figures that have the same center are called *concentric*.

concurrent: Three (or more) lines that go through a common point are *concurrent*. [212]

concyclic: Points that all lie on a single circle are called *concyclic*. [691]

congruent: When the points of one figure can be matched with the points of another figure, so that corresponding parts have the same size, then the figures are called *congruent*, which means that they are considered to be equivalent. [19, 82, 97, 98]

converse: The converse of a statement of the form "if [something] then [something else]" is the statement "if [something else] then [something]." [340]

convex: A polygon is called *convex* if every segment joining a pair of points within it lies entirely within the polygon. [378]

coordinates: Numbers that describe the position of a point in relation to the origin of a coordinate system.

corresponding: Describes parts of figures (such as angles or segments) that have been matched by means of a transformation. [98]

cosine ratio: Given a right triangle, the cosine of one of the acute angles is the ratio of the length of the side *adjacent* to the angle to the length of the hypotenuse. The word cosine is a combination of *complement* and *sine*, so named because the cosine of an angle is the same as the sine of the complementary angle. [572, 622]

CPCTC: *Corresponding Parts of Congruent Triangles are themselves Congruent.* [page 18]

Crossed-Chords Theorem: When two chords intersect inside a circle, the product of the lengths of the segments of one chord equals the product of the lengths of the segments of the other chord. Thus the value of this product depends on only the location of the point of intersection. [771, 783]

cyclic: A polygon, all of whose vertices lie on the same circle, is called *cyclic*. Also called an *inscribed polygon*. [727, 728, 737, 749]

decagon: A polygon that has ten sides. [447]

diagonal: A segment that connects two nonadjacent vertices of a polygon. [89]

dialation: There is no such word. See *dilation*.

diameter: A chord that goes through the center of its circle is called a *diameter*. [640]

dihedral: An angle that is formed by two intersecting *planes*. To measure its size, choose a point that is common to both planes, then through this point draw the line in each plane that is perpendicular to their line of intersection. [651, 665, 767, 791, 814]

dilation: A similarity transformation, with the special property that all lines obtained by joining points to their images are concurrent at the same *central* point. [554, 561, 567, 574]

directed segment: A segment in the coordinate plane with an initial and terminal point. [67]

direction vector: A vector that describes a line, by pointing from a point on the line to some other point on the line. [188]

directrix: See *parabola*.

displacement vector: The displacement vector from point (a, b) to point (c, d) is the vector $[c - a, d - b]$. [93, 101]

distance formula: The distance from (x_1, y_1) to (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, and the distance from (x_1, y_1, z_1) to (x_2, y_2, z_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$. These formulas are consequences of the *Pythagorean Theorem*.

dodecagon: A polygon that has twelve sides. [484]

dodecahedron: A polyhedron formed by attaching twelve polygons edge to edge. If the dodecagon is regular, each of its vertices belongs to three congruent *regular* pentagons.

Doppler shift: The change of frequency that results when the source of a signal is moving relative to the observer. [503]

dot product: Given vectors $[a, b]$ and $[m, n]$, their dot product is the number $am + bn$. Given vectors $[a, b, c]$ and $[p, q, r]$, their dot product is the number $ap + bq + cr$. In either case, it is the sum of the products of corresponding components. When the value is zero, the vectors are perpendicular, and conversely. [210, 314, 321]

equiangular: A polygon all of whose angles are the same size. [12]

equidistant: A shortened form of *equally distant*. [14]

equilateral: A polygon all of whose sides have the same length. [5]

Euclidean geometry (also known as plane geometry) is characterized by its parallel postulate, which states that, *given a line, exactly one line can be drawn parallel to it through a point not on the given line.* A more familiar version of this assumption states that *the sum of the angles of a triangle is a straight angle.* [26, 318]The Greek mathematician Euclid, who flourished about 2300 years ago, wrote many books, and established a firm logical foundation for geometry.

Euler line: The centroid, the circumcenter, and the orthocenter of any triangle are collinear. [253]The Swiss scientist Leonhard Euler (1707-1783) wrote copiously on both mathematics and physics, and knew the *Aeneid* by heart.

exterior angle: An angle that is formed by a side of a polygon and the extension of an adjacent side. It is supplementary to the adjacent interior angle. [313, 322, 323, 324, 325]

Exterior-Angle Theorem: An exterior angle of a triangle is the sum of the two nonadjacent interior angles. [325, 474]

extrapolation: After having used a set of data to find a least squares line, extrapolation is the process of finding a point on the least squares line where the x-value is outside the range of the x-values of the data set. [697]

focus: See *parabola*.

foot: The point where an altitude meets the base to which it is drawn. [173, 615, 816]

function: A function is a rule that describes how an input uniquely determines an output. [9, 40, 94, 109]

geometric mean: If x and y are positive numbers, $\sqrt{xy}$ is their *geometric mean.* Note that the geometric mean is different from the arithmetic mean and the harmonic mean. [813, 820]

glide-reflection: An isometric transformation of a plane that leaves no single point fixed, but that does map a single line to itself. This transformation consists of a translation (a glide) and a reflection, in which the translation vector is parallel to the line of reflection. The order of transformations does not matter. [84, 200]

Greek letters appear often in mathematics. Some of the common ones are α (alpha), β (beta), Δ or δ (delta), θ (theta), Λ and λ (lambda), μ (mu), π (pi), and Ω or ω (omega). [465, 716]

harmonic mean: The reciprocal of the arithmetic mean of the reciprocals. The *harmonic mean* of a and b is $\dfrac{1}{\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}\right)}$. Note that the harmonic mean is different from the arithmetic mean and the geometric mean. [722, 763, 813]

head: Vector terminology for the terminal point of a directed segment. [67]

hexagon: a polygon that has six sides. [12]

Hypotenuse-Leg: When the hypotenuses of two right triangles have the same length, and a leg of one triangle has the same length as a leg of the other, then the triangles are congruent. This rule of evidence is abbreviated to HL. [265]

icosahedron: A polyhedron formed by attaching twenty polygons edge to edge. If the polyhedron is regular, each of its vertices belongs to five equilateral triangles. [195]

icosidodecahedron: A polyhedron formed by attaching the edges of twenty equilateral triangles to the edges of twelve regular pentagons. Two triangles and two pentagons meet at each vertex. [265]

image: The result of applying a transformation to a point P is called the *image point* of P and is often denoted P'. One occasionally refers to an *image segment* or an *image triangle*. [74, 75, 185]

incenter: The angle bisectors of a triangle are concurrent at this point, which is equidistant from the sides of the triangle. [754]

included angle: The angle formed by two designated segments. [726]

inscribed angle: An angle formed when two chords meet at a point on the circle. An inscribed angle is *half* the angular size of the arc it intercepts. In particular, an angle that intercepts a semicircle is a *right* angle. [671, 672]

inscribed polygon: A polygon whose vertices all lie on the same circle; also called a *cyclic polygon*. [670, 690, 727, 728]

integer: Any whole number, whether it be positive, negative, or zero. [25]

intercepted arc: The part of an arc that is found inside a given angle. [671]

interpolation: After having used a set of data to find a least squares line, interpolation is the process of finding a point on the least squares line where the x-value is inside the range of the x-values of the data set. [697]

isometry: A geometric transformation that preserves distances. The best-known examples of isometries are *translations*, *rotations*, and *reflections*. [204]

isosceles triangle: A triangle that has two sides of the same length. [21]The word is derived from the Greek *iso* + *skelos* (equal + leg)

Isosceles-Triangle Theorem: If a triangle has two sides of equal length, then the angles opposite those sides are also the same size. [202]

isosceles trapezoid: A trapezoid whose nonparallel sides have the same length. [404]

kite: A quadrilateral that has two disjoint pairs of congruent adjacent sides, or, equivalently, a quadrilateral that has reflective symmetry across a *diagonal*. [16, 171]

labeling convention: Given a polygon that has more than three vertices, place the letters around the figure in the order that they are listed. [181, 186, 231]

lateral face: Any face of a pyramid or prism that is not a base. [765]

lattice point: A point whose coordinates are both integers. [25]

lattice rectangle: A rectangle whose vertices are all lattice points. [145]

least squares line: Given a data set, the least squares line is the line through the centroid which minimizes the sum of the squares of the residuals. [637]

leg: The perpendicular sides of a right triangle are called its legs. [59]

length of a vector: This is the length of any segment that represents the vector. [67, 86]

lens: A region enclosed by two intersecting, non-concentric circular arcs. [789]

linear equation: Any straight line can be described by an equation in the form $ax+by=c$. [31, 37]

linear regression: Given a data set, a process which finds the least squares line for that set of data. [637]

lower: A term used at Phillips Exeter Academy to refer to a tenth grader. Historically the term *lower middler* has also been used. [473]

magenta: A shade of purple, named for a town in northern Italy. [674]

magnitude of a dilation: The nonnegative number obtained by dividing the length of any segment into the length of its dilated image. See *ratio of similarity*. [567, 583]

major/minor arc: Two arcs are determined by a given chord. The smaller arc is called *minor*, and the larger arc is called *major*. [641]

MasterCard: [743]

median of a triangle: A segment that joins a vertex of a triangle to the midpoint of the opposite side. [184]

midline of a trapezoid: This segment joins the midpoints of the non-parallel sides. Its length is the average of the lengths of the parallel sides, to which it is also parallel. Also known as the *median* in some books. [494]

Midline Theorem: A segment that joins the midpoints of two sides of a triangle is parallel to the third side, and is half as long. [373, 450]

midpoint: The point on a segment that is equidistant from the endpoints of the segment. If the endpoints are (a, b) and (c, d), the midpoint is $\left(\dfrac{a+c}{2}, \dfrac{b+d}{2}\right)$. [30]

mirror: See *reflection.*

negative association: A negative association between two variables is a relationship such that when one variable increases, the other decreases. [747]

negative reciprocal: One number is the negative reciprocal of another if the product of the two numbers is -1. [27]

normal vector: In 2 dimensions, a normal vector is a vector perpendicular to a line. In 3 dimensions, a normal vector may also refer to a vector that is perpendicular to a line, a plane, or even a curved surface. [190]

octagon: a polygon that has eight sides. [285]

opposite: Two numbers or vectors are opposite if they differ in sign. For example, 17.5 is the opposite of -17.5, and $[2, -11]$ is the opposite of $[-2, 11]$. [86, 104]

opposite angles: In a quadrilateral, this means non-adjacent angles. [341]

opposite sides: In a quadrilateral, this means non-adjacent sides. [135]

orthocenter: The altitudes of a triangle are concurrent at this point. [212, 512]

parabola: A curve consisting of those points that are equidistant from a given line and a given point form a curve called a *parabola.* The given point is called the *focus* and the given line is called the *directrix.* The point on the parabola that is closest to the directrix (thus closest to the focus) is the *vertex.* [332, 463, 464, 465, 638, 663]

parallel: Coplanar lines that do not intersect. When drawn in a coordinate plane, they are found to have the same slope, or else no slope at all. The shorthand $\|$ is often used. [38]

parallelogram: A quadrilateral that has two pairs of parallel sides. [135]

parameter: See *parametric equations.* [70]

parametric equations: Equations that determine 2D and 3D coordinates in terms of a separate variable, typically t, which is referred to as a *paramter.* [70]

pentagon: a polygon that has five sides. [195, 246]

perpendicular: Coplanar lines that intersect to form a right angle. If m_1 and m_2 are the slopes of two lines in the xy-plane, neither line parallel to a coordinate axis, and if $m_1 m_2 = -1$, then the lines are perpendicular. [26]

perpendicular bisector: Given a line segment, this is the line that is perpendicular to the segment and that goes through its *midpoint*. The points on this line are all *equidistant* from the endpoints of the segment. [31]

perpendicular vectors: Two vectors whose dot product is zero. [210, 536]

point-slope form: A non-vertical straight line can be described by $y - y_0 = m(x - x_0)$ or by $y = m(x - x_0) + y_0$. One of the points on the line is (x_0, y_0) and the slope is m. [38]

polygon: A closed 2D figure made up of line segments, called *edges* that meet at their endpoints. The points where the segments meet are called *vertices* and the number of edges and vertices are equal. Note that the definition of a polygon is one of many places in which not all mathematicians agree.

postulate: A statement that is accepted as true, without proof. [26]

prep: A term used at Phillips Exeter Academy to refer to a ninth grader. Historically the term *juniors* has also been used. [51]

prism: A three-dimensional figure that has two congruent and parallel *bases*, and parallelograms for its remaining *lateral faces*. If the lateral faces are all rectangles, the prism is a *right prism*. If the base is a regular polygon, the prism is also called *regular*. [286, 814]

probability: A number between 0 and 1, often expressed as a percent, that expresses the likelihood that a given event will occur. For example, the probability of rolling a 2 with a standard 6-sided die is $\frac{1}{6}$ or approximately 16.67%. [811, 813, 813]

proportion: An equation that expresses the equality of two ratios. [553, 681]

pyramid: A three-dimensional figure that is obtained by joining all the points of a polygonal *base* to a *vertex*. Thus all the lateral faces of a pyramid are triangles. If the base polygon is regular, and the lateral edges are all congruent, then the pyramid is called *regular*. [765]

Pythagorean Theorem: The square on the hypotenuse of a right triangle equals the sum of the squares on the legs. If a and b are the lengths of the legs of a right triangle, and if c is the length of the hypotenuse, then these lengths fit the Pythagorean equation $a^2 + b^2 = c^2$. [5, 14] Little is known about the Greek figure Pythagoras, who flourished about 2500 years ago, except that he probably did not discover the theorem that bears his name.

QED: QED and □ are typical ways to indicate a proof is complete. QED is an abbreviation for the latin “quod erat demonstrandum” which means “that which was to be demonstrated. [170, 171]

quadrant: one of the four regions formed by the coordinate axes. Quadrant I is where both coordinates are positive, and the other quadrants are numbered (using Roman numerals) in a counterclockwise fashion. [14]

quadratic formula: $x = \dfrac{-b+\sqrt{b^2-4ac}}{2a}$ and $x = \dfrac{-b-\sqrt{b^2-4ac}}{2a}$ are the two solutions to $ax^2+bx+c=0$.

quadrilateral: a four-sided *polygon*. [5, 13, 78, 89]

ray: A semi-infinite line, which starts at a point and extends infinitely far in only one direction. If a ray starts at point A, passes through point B, and the continues to infinity, we call the ray "AB".

radial expansion: See *dilation*.

ratio of similarity: The ratio of the lengths of any two corresponding segments of *similar* figures. [573]

reflection: An isometric transformation of a plane that has a line of fixed points. A reflection maps points on either side of this line (the *mirror*) to the other side. [84, 200]

reflection property of a parabola: Through any point on a *parabola*, draw the line that is parallel to the axis of symmetry, the line that goes through the focus, and the tangent line. The first two lines make equal angles with the third. [652]

regular: A polygon that is both equilateral and equiangular. [246]

regular pyramid: See *pyramid*.

residual: Given a line $y = mx + b$ and a point (x_1, y_1) not on the line, the difference $y_1 - (mx_1 + b)$ is called a *residual*. Its magnitude is the vertical distance between the point and the line. Its sign tells whether the point is above or below the line. [597]

Rhode Island School of Design.[p. 86]

rhombus: An equilateral quadrilateral. [5, 89]

right angle: An angle that is its own supplement. [26, 172]

rotation: An isometric transformation of a plane that leaves a single point fixed. [84, 200]

SAS similarity: Two triangles are certain to be similar if two sides of one triangle are proportional to two sides of the other, and if the included angles are equal in size. [726]

scalar: In the context of vectors, this is just another name for a number. [105]

scalene: A triangle no two of whose sides are the same length. [218]

scatterplot: Given a set of data, a scatter plot is a two-dimensional graph where the coordinates of the point represent the corresponding values of the two variables in the data set. [608]

segment: That part of a line that lies between two designated points. [1, 27, 30]

Sentry Theorem: The sum of the exterior angles (one at each vertex) of any polygon is 360 degrees. [325, 364, 372, 474]

Shared-Altitude Theorem: If two triangles share an altitude, then the ratio of their areas is proportional to the ratio of the corresponding bases. [555]

Shared-Base Theorem: If two triangles share a base, then the ratio of their areas is proportional to the ratio of the corresponding altitudes. [581]

Side-Angle-Side: When the parts of one triangle can be matched with the parts of another triangle, so that two pairs of corresponding sides have the same lengths, and so that the (corresponding) angles they form are also the same size, then the triangles are congruent. This rule of evidence is abbreviated to just SAS. [90]

Side-Side-Angle: This is insufficient evidence for congruence. [96, 261] See the item *Hypotenuse-Leg*, however.

Side-Side-Side: When the parts of one triangle can be matched with the parts of another triangle, so that all three pairs of corresponding sides have the same lengths, then the triangles are congruent. This rule of evidence is abbreviated to just SSS. [82]

similar: Two figures are similar if their points can be matched in such a way that all ratios of corresponding lengths are proportional to a fixed *ratio of similarity.* [573]

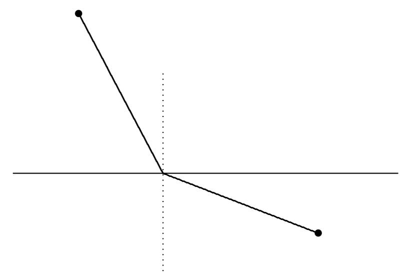
sine ratio: Given a right triangle, the sine of one of the acute angles is the ratio of the length of the side *opposite* the angle to the length of the hypotenuse. [559, 565]

skew lines: Non-intersecting lines whose direction vectors are not parallel. [614]

slope: The slope of the segment that joins the points (x_1, y_1) and (x_2, y_2) is $\dfrac{y_2 - y_1}{x_2 - x_1}$. [27, 32, 48]

slope-intercept form: Any non-vertical straight line can be described by an equation that takes the form $y = mx + b$. The slope of the line is m, and the y-intercept is b.

Snell's Law: Also known as the *Law of Refraction*, this describes the change in direction that occurs when light passes from one medium to another. The ratio of speeds is equal to the ratio of the sines of the angles formed by the rays and lines perpendicular to the interface. The Dutch physicist Willebrod Snell (1580-1626) did not tell anyone of this discovery when he made it in 1621. [805, 805]



SSS similarity: Two triangles are *similar* if their sides are proportional. [620]

standard form: Standard form of a linear equation refers the form $ax+by=c$. This should not be confused with standard form of a quadratic function, $y = ax^2 + bx + c$, where $a \neq 0$. [190]

stop sign: [285]

subtended angle: Given a point O and a figure $\mathcal{F}$, the *angle subtended by* $\mathcal{F}$ *at* O is the smallest angle whose vertex is O and whose interior contains $\mathcal{F}$. [654, 819, 820]

supplementary: Two angles that fit together to form a straight line are called *supplementary*. Each angle is the *supplement* of the other. [26]

symmetry axis of a parabola: The line through the focus that is perpendicular to the directrix. Except for the *vertex*, each point on the parabola is the reflected image of another point on the parabola. [666]

tail: Vector terminology for the initial point of a directed segment. [67]

tail-to-tail: Vector terminology for directed segments with a common first vertex. [283]

tangent ratio: Given a right triangle, the tangent of one of the acute angles is the ratio of the side opposite the angle to the side adjacent to the angle. [453, 454, 455, 633]

tangent and slope: When an angle is formed by the positive x-axis and a ray through the origin, the *tangent* of the angle is the *slope* of the ray. Angles are measured in a counterclockwise sense, so that rays in the second and fourth quadrants determine negative tangent values. [453]

tangent to a circle: A line that touches a circle without crossing it. Such a line is perpendicular to the radius drawn to the point of tangency. [746]

tangent to a parabola: A line that intersects the curve without crossing it. To draw the tangent line at a given point on a parabola, join the nearest point on the directrix to the focus, then draw the perpendicular bisector of this segment. [307, 308, 309, 465, 639, 652]

tesselate: To fit non-overlapping tiles together to cover a planar region. [371, 371, 378, 452]

tetrahedron: A pyramid whose four faces are all triangles. [797]

Three-Parallels Theorem: Given three parallel lines, the segments they intercept on one transversal are proportional to the segments they intercept on any transversal. [449, 459]

transformation: A *function* that maps points to points. [75, 78, 84, 97, 154, 160, 200]

translate: To slide a figure by applying a vector to each of its points. [74, 200]

transversal: A line that intersects two other lines in a diagram. [313, 316]

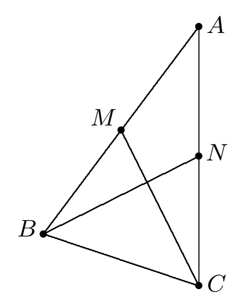
trapezoid: A quadrilateral with exactly one pair of parallel sides. If the non-parallel sides have the same length, the trapezoid is called *isosceles*. [404]

triangle inequality: For any P, Q, and R, $PQ \le PR + RQ$. It says that any side of a triangle is less than or equal to the sum of the other two sides. [116, 422]

truncated icosahedron: A polyhedron obtained by slicing off the vertices of an icosahedron. The twelve icosahedral vertices are replaced by twelve pentagons, and the twenty icosahedral triangles become twenty hexagons. [195]

two-column proof: A way of outlining a geometric deduction. Steps are in the left column, and supporting reasons are in the right column. For example, here is how one might show that an isosceles triangle ABC has two medians of the same length. It is given that $AB = AC$ and that M and N are the midpoints of sides AB and AC, respectively. The desired conclusion is that medians CM and BN have the same length. [170]

$AB = AC$	given
$AM = AN$	M and N are midpoints
$\angle MAC = \angle NAB$	shared angle
$\Delta MAC \cong \Delta NAB$	SAS
$CM = BN$	CPCTC



Two-Tangent Theorem: From a point outside a circle, there are two segments that can be drawn tangent to the circle. These segments have the same length. [773]

unit circle: This circle consists of all points that are 1 unit from the origin O of the xy-plane. Given a point P on this circle, the coordinates of P are the *cosine* and the *sine* of the counterclockwise angle formed by segment OP and the positive x-axis. [670]

unit square: Its vertices are $(0, 0)$, $(1, 0)$, $(0, 1)$, and $(1, 1)$. [274]

upper: A term used at Phillips Exeter Academy to refer to an eleventh grader. Historically the term *upper middler* has also been used. [489]

Varignon parallelogram: Given any quadrilateral, this is the figure formed by connecting the midpoints of consecutive sides [514]. The French mathematician Pierre Varignon (1654-1722) learned calculus when it was a new science, then taught it to others.

vectors have *magnitude* (size) and *direction*. Visualize them as directed segments (arrows). Vectors are described by *components*, just as points are described by coordinates. The vector from point A to point B is often denoted $\overrightarrow{AB}$, or abbreviated by a boldface letter such as $\mathbf{u}$, and its magnitude is often denoted $\left|\overrightarrow{AB}\right|$ or $|\mathbf{u}|$. [67, 104, 105, 106, 146, 303, 304, 311]

vector triangles: Given vectors $\mathbf{u} = [a, b]$ and $\mathbf{v} = [c, d]$, a triangle is determined by drawing $\mathbf{u}$ and $\mathbf{v}$ so that they have a common initial point (*tail-to-tail*). No matter what this initial point is, the triangles determined by $\mathbf{u}$ and $\mathbf{v}$ are all congruent. All have $\frac{1}{2}|ad - bc|$ as their area. [238]

velocity: A vector obtained by dividing a displacement vector by the elapsed time.[226]

vertex of a parabola: The point on a parabola that is closest to the focus. [463, 666]

vertex of a polygon: The point at which two sides meet in a polygon. The plural is *vertices*, but "vertice" is not a word. [21, 49, 74, 79]

vertical angles: Puzzling terminology that is often used to describe a pair of nonadjacent angles formed by two intersecting lines. [126]

volume of a prism: This is the product of the *base area* and the *height*, which is the distance between the parallel base planes.

volume of a pyramid: This is one third of the product of the *base area* and the *height*, which is the distance from the vertex to the base plane.

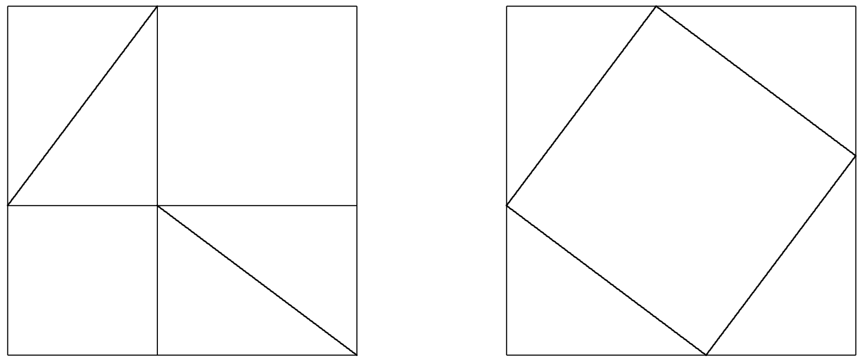
volumes of similar figures: If two three-dimensional figures are similar, then the ratio of their volumes equals the *cube* of the ratio of similarity.

zero-residual line: Given a data set, a zero-residual line is a line such that the sum of the residuals of the data set with respect to the line is zero. [608]

Math 2 Appendix

5-1. The main use of the Pythagorean Theorem is to find distances. Originally (6^{th} century BC), however, it was regarded as a statement about *areas*. Explain this interpretation.

14-1. The two-part diagram below, which shows two different dissections of the same square, was designed to help *prove* the Pythagorean Theorem. Provide the missing details.



35-1. (Continuation) Assume that Pat and Kim run at p and k miles per hour, respectively, and that Pat finishes m minutes before Kim. Find the length of the race, in miles.

37-1. Rewrite the equation $3x - 5y = 30$ in the form $ax + by = 1$. Are there lines whose equations *cannot* be rewritten in this form?

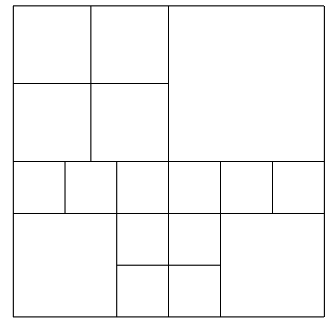
59-1. Lynn takes a step, measures its length and obtains 3 feet. Lynn uses this measurement in attempting to pace off a 1-mile course, but the result is 98 feet too long. What is the actual length of Lynn's stride, and how could Lynn have done a more accurate job?

59-2. One of the legs of a right triangle is 12 units long. The other leg is b units long and the hypotenuse c units long, where b and c are both integers. Find b and c. *Hint*: Both sides of the equation $c^2 - b^2 = 144$ can be factored.

65-1. Golf balls cost \$0.90 each at a local golf club, which has an annual \$25 membership fee. At the sporting-goods store down the road, the price is \$1.35 per ball for the same brand. Where you buy your golf balls depends on how many you wish to buy. Explain, and illustrate your reasoning by drawing a graph.

67-1. Is it possible for a positive number to exceed its reciprocal by exactly 1? One number that comes close is $\frac{8}{5}$, because $\frac{8}{5} - \frac{5}{8}$ is $\frac{39}{40}$. Is there a fraction that comes closer?

Math 2 Appendix



75 -1. It is a simple matter to divide a square into four smaller squares, and — as the figure at right shows — it is also possible to divide a square into seventeen smaller squares. In addition to four and seventeen, what numbers of smaller squares are possible? The smaller squares can be of any size whatsoever, as long as they fit neatly together to form one large square.

84 -1. In baseball, the bases are placed at the corners of a square whose sides are 90 feet long. Home plate and second base are at opposite corners. To the nearest eighth of an inch, how far is it from home plate to second base?

87 -1. A 9-by-12 rectangular picture is framed by a border of uniform width. Given that the combined area of picture plus frame is 180 square units, find the width of the border.

99 -1. A debt of \$450 is to be shared equally among the members of the Outing Club. When five of the members refuse to pay, the other members' shares each go up by \$3. How many members does the Outing Club have?

111 -1. A puzzle: Cut out four copies of the quadrilateral $ABCD$ formed by points $A = (0, 0)$, $B = (5, 0)$, $C = (6, 2)$, and $D = (0, 5)$. Show that it is possible to arrange these four pieces to form a square. Explain why you are sure that the pieces fit exactly.

131 -1. If I were to increase the length of my stride by one inch, it would take me 60 fewer strides to cover a mile. What was the length of my original stride?

140 -1. The sides of a right triangle are $x - y$, x, and $x + y$, where x and y are positive numbers, and $y < x$. Find the ratio of x to y.

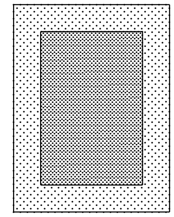
140 -2. After taking the Metro to Dupont Circle in Washington, DC, Jess reached street level by walking up the escalator at a brisk rate, taking 42 steps during the trip to the top. Suddenly curious about the length of the escalator, Jess returned to the bottom and walked up the same escalator at a leisurely rate, taking steps one half as often as on the first trip, taking 24 steps in all. How many steps can be seen on the visible part of the escalator?

162 -1. The triangle with vertices $(0, 0)$, $(2, 1)$, and $(0, 5)$ can be cut into pieces that are each congruent to the triangle with vertices $(2, 0)$, $(3, 0)$, and $(3, 2)$. Show how.

173 -1. If I were to increase my cycling speed by 3 mph, I calculate that it would take me 40 seconds less time to cover each mile. What is my current cycling speed?

182 -1. Make up a geometry problem to go with the equation $x + 3x + x\sqrt{10} = 42$.

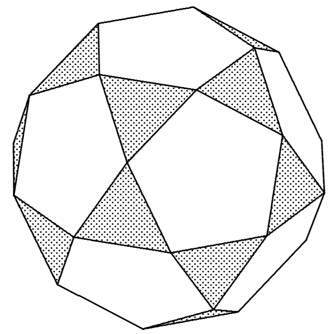
Math 2 Appendix



203 -1. Robin is mowing a rectangular field that measures 24 yards by 32 yards, by pushing the mower around and around the outside of the plot. This creates a widening border that surrounds the unmowed grass in the center. During a brief rest, Robin wonders whether the job is half done yet. How wide is the uniform mowed border when Robin *is* half done?

259 -1. Given that $ABCDEFGHI$ is a regular polygon, prove that AD and FI have the same length.

264 -1. Consider the following process for bisecting an angle ABC: First mark M on BA and P on BC so that $MB = PB$, then mark new points N on BA and Q on BC so that $NB = QB$. Let E be the intersection of MQ and NP. Prove that segment BE is the desired angle bisector.



265 -1. An *icosidodecahedron* has twelve pentagonal faces, as shown at right. How many edges does this figure have? How many vertices? How many triangular faces?

270 -1. Suppose that triangle PAB is isosceles, with $AP = PB$, and that C is on side PB, between P and B. Show that $CB < AC$.

271 -1. Draw the lines $y = 0$, $y = \frac{1}{2}x$, and $y = \frac{4}{3}x$. Use your protractor to measure the angles, then make calculations to confirm what you observe.

346 -1. Although you have used the converse of the Pythagorean Theorem, it has not yet been proved in this book. State and prove the converse.

355 -1. In what sense is a transformation a function?

371 -1. Let $A = (1, 1)$, $B = (3, 5)$, and $C = (7, 2)$. Explain how to cover the whole plane with non-overlapping triangles, each of which is congruent to triangle ABC.

371 -2. (Continuation) In the pattern of lines produced by your *tesselation*, you should see triangles of many different sizes. What can you say about their sizes and shapes?

375 -1. Draw an 8-by-9-by-12 box $ABCDEFGH$. How many right triangles can be formed by connecting three of the eight vertices?

378 -1. Is it possible for a pentagon to have interior angles 120°, 120°, 120°, 90°, and 90°, *in this order*? What about 120°, 120°, 90°, 120°, and 90°? Are there other arrangements of the five angles that could have been considered? Do any of these pentagons *tesselate*?

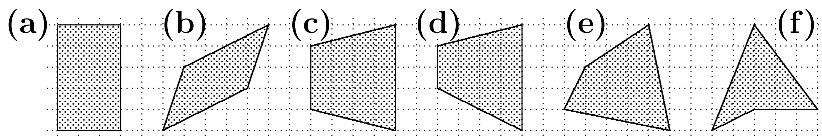
386 -1. Can two of the angle bisectors of a triangle intersect perpendicularly? Explain.

432 -1. Is it possible for a trapezoid to have sides of lengths 3, 7, 5, and 11?

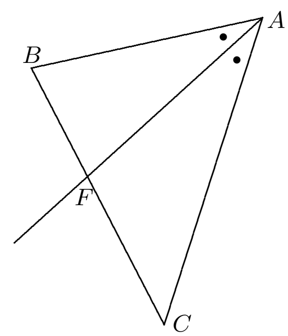
Math 2 Appendix

449 -1. Let $A = (0,0)$, $B = (6,2)$, $C = (-5,2)$, $D = (7,6)$, $E = (-1,7)$, and $F = (5,9)$. Draw lines AB, CD, and EF. Verify that they are parallel.
(a) Draw the transversal of slope -1 that goes through E. This transversal intersects line AB at G and line CD at H. Use your ruler to measure EH and HG.
(b) Draw any transversal whose slope is 2 (it need not go through E), intersecting line AB at P, line CD at Q, and line EF at R. Use your ruler to measure PQ and QR.
(c) On the basis of your findings, propose a theorem. A proof is not requested.

452 -1. Which of these quadrilaterals can be used to tesselate a plane? Justify your choices.



470 -1. (Continuation) Draw an acute-angled, scalene, lattice triangle ABC of your choosing, then use your protractor to carefully draw the bisector of angle BAC. Let F be the intersection of the bisector with BC. Measure the lengths AB, AC, FB, and FC. Do you notice anything?



472 -1. A rectangular box is 2 by 3 by h, and two of its internal diagonals are perpendicular. Find possible values for h.

477 -1. Suppose that $ANGEL$ is a regular pentagon, and that $CANT$, $HALF$, $ROLE$, $KEGS$, and $PING$ are squares attached to the outside of the pentagon. Show that decagon $PITCHFORKS$ is equiangular. Is this decagon equilateral?

478 -1. Rearrange the letters of *doctrine* to spell a familiar mathematical word.

484 -1. Suppose that $ASCENT$ is a regular hexagon, and that $ARMS$, $BATH$, $LINT$, $FEND$, $COVE$, and $CUPS$ are squares attached to the outside of the hexagon. Decide whether or not dodecagon $LIDFVOUPMRBH$ is regular, and give your reasons.

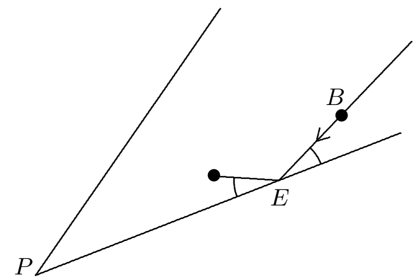
501 -1. Let $ABCD$ be a square. Mark midpoints M, N, O, and P on AB, BC, CD, and DA, respectively. Draw AN, BO, CP, and DM. Let Q and R be the intersections of AN with DM and BO, respectively, and let S and T be the intersections of CP with BO and DM, respectively. Prove as much as you can about this figure, especially quadrilateral $QRST$.

501 -2. (Continuation) Segment AB is 10 cm long. How long is QR, to the nearest 0.1 cm?

503 -1. *The Doppler Shift*. While driving a car, AJ honks the horn every 5 seconds. Hitch is standing by the side of the road, and hears the honks of the oncoming car every 4.6 seconds. The speed of sound is 330 meters per second. Calculate the speed of AJ's car. Describe what Hitch hears after the car passes.

Math 2 Appendix

508 -1. The diagram at right shows one corner of a triangular billiards table. A ball leaves point B and follows the indicated path, striking the edge of the table at E. Thereafter, at each impact, the ball obeys the law of reflection, which says that the incoming angle equals the outgoing angle. Given that there is a 34° angle at corner P, and that the initial impact makes a 25° angle at E, how many bounces will the ball make before its path leaves this page?



521 -1. Show that a regular dodecagon can be cut into pieces that are all regular polygons, which need not all have the same number of sides.

525 -1. Alden, a passenger on a yacht moored 15 miles due north of a straight, east-west coastline, has become ill and has to be taken ashore in a small motorboat, which will meet an ambulance at some point on the shore. The ambulance will then take Alden to the hospital, which is 60 miles east of the shore point closest to the yacht. The motorboat can travel at 20 mph and the ambulance at 90 mph. In what direction should the motorboat head, to minimize the travel time to the hospital? Express your answer using an angle.

536 -1. A regular n-sided polygon has exterior angles of m degrees each. Express m in terms of n. For how many of these regular examples is m a whole number?

548 -1. The diameter of the earth's orbit around the sun is approximately 186 million miles. Looking at a star from the two points on the orbit which are furthest apart, the lines of sight to the star form an angle of 4.269×10^{-4} degrees. How many light-years away is this star from the earth? Note that one light-year is approximately 5.879×10^{12} miles.

557 -1. To the nearest tenth of a degree, find the sizes of the acute angles in the 7-24-25 right triangle and in the 8-15-17 right triangle. This information then allows you to calculate the sizes of all the angles in the 25-51-52 triangle. Show how to do it.

560 -1. If you were to drop a ball from a height of 50 feet, how high would it bounce? To make such a prediction, you could gather data by experimenting with smaller heights, where it is easier to measure the rebound. Gather several data points (drop, rebound), using a meter stick and a sufficiently bouncy ball. If you use the top of the ball for your measurements, remember to take the diameter of the ball into account when recording your data.

580 -1. Draw a large acute-angled triangle ABC. Carefully add the altitudes AE and BF to your drawing. Measure the lengths of AE, BF, BC, and AC. Where have you seen the equation $(AE)(BC) = (BF)(AC)$ before? What can you say about the right triangles AEC and BFC? Justify your response.

Math 2 Appendix

608 -1. We have shown in a previous problem that the coordinates of the centroid of a triangle can be found by averaging the x- and y-values of the three vertices. In general, the centroid of a set of points is $(\bar{x}, \bar{y})$, where $\bar{x}$ is the average of the x-coordinates and $\bar{y}$ is the average of the y-coordinates.
(a) Make a *scatterplot* of the points $(1, -2)$, $(3, 3)$, $(4, 0)$, $(7, 1)$, and $(10, 8)$. Also calculate and plot the centroid of this set of points.
(b) Write an equation of the line with slope 1 containing the centroid, and graph this on the scatterplot. Find the sum of the residuals for the five points in the data set.
(c) Repeat part (b) with a slope of -1, and again with a slope of 5.
(d) Formulate a conjecture about the sum of the residuals for a line that contains the centroid of a data set.

608 -2. (Continuation) On the previous scatterplot, graph the line with equation $y = 0.88x - 2.4$. Confirm that this is a *zero-residual line*. How does this line seem to fit the scatterplot? Among the family of lines that are zero-residual lines, this line is the best fit in the sense that it minimizes the sum of the squares of the residuals.

639 -1. Suppose that one of the angles of a triangle is exactly twice the size of another angle of the triangle. Show that *any* such triangle can be dissected, by a single straight cut, into two triangles, one of which is isosceles, the other of which is similar to the original.

641 -1. For their students who turn the steering wheel too often while on the freeway, driving instructors suggest that it is better to focus on a point that is about 100 yards ahead of the car than to focus on a point only 10 yards ahead of the car. Comment on this advice.

644 -1. Let A and B be the positive x-intercept and the positive y-intercept, respectively, of the circle $x^2 + y^2 = 18$. Let P and Q be the positive x-intercept and the positive y-intercept, respectively, of the circle $x^2 + y^2 = 64$. Verify that the ratio of chords $AB : PQ$ matches the ratio of the corresponding diameters. What does this data suggest to you?

646 -1. Make an accurate drawing of a regular hexagon $ABCDEF$. Be prepared to report on the method you used to draw this figure. Measure the length of diagonal AC and the length of side AB. Form the *ratio* of the diagonal measurement to the side measurement. When you compare answers with your classmates, on which of these three numbers do you expect to find agreement?

646 -2. (Continuation) Calculate $AC : AB$. One way to do it is to use trigonometry.

646 -3. (Continuation) The diagonals AC, BD, CE, DF, EA, and FB form the familiar six-pointed *Star of David*. Their intersections inside $ABCDEF$ are the vertices of a smaller hexagon. Explain why the small hexagon is similar to $ABCDEF$. In particular, explain why the small hexagon is regular. Make measurements and use them to approximate the *ratio of similarity*. Then calculate an exact value for this ratio.

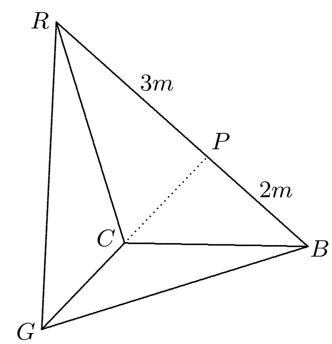
Math 2 Appendix

674 -1. A close look at a color television screen reveals an array of thousands of tiny red, green, and blue dots. This is because any color can be obtained as a *mixture* of these three colors. For example, if neighboring red, green, and blue dots are equally bright, the effect is white. If a blue dot is unilluminated and its red and green neighbors are equally bright, the effect is yellow. In other words, white corresponds to the red:green:blue ratio $\frac{1}{3}:\frac{1}{3}:\frac{1}{3}$ and pure yellow corresponds to $\frac{1}{2}:\frac{1}{2}:0$. Notice that the sum of the three terms in each proportion is 1. A triangle RGB provides a simple model for this mixing of colors. The vertices represent three neighboring dots. Each point C inside the triangle represents a precise color, defined as follows: The *intensities* of the red dot, green dot, and blue dot are proportional to the *areas* of the triangles CGB, CBR, and CRG, respectively. What color is represented by the centroid of RGB? What color is represented by the midpoint of side RG?

674 -2. (Continuation) Point C is $\frac{3}{5}$ of the way from R to G. Give a numerical description for the color mixture that corresponds to it. The color *magenta* is composed of equal intensities of red and blue, with green absent. Where is this color in the triangle?

674 -3. (Continuation) Given that color C is defined by the red:green:blue ratio $0.4:g:b$, where $g+b=0.6$, what are the possible positions for C in the triangle?

678 -1. Given SSS information about an isosceles triangle, describe the process you would use to calculate the sizes of its angles.



682 -1. In triangle RGB, mark P on side RB so that $RP:PB$ equals $3:2$. Let C be the midpoint of GP. Calculate the ratio of areas $CGB:CBR:CRG$. Express your answer

(a) so that the sum of the three numbers is 1;
(b) so that the three numbers are all integers.

682 -2. Mixtures of *three* quantities can be modeled geometrically by using a triangle. What geometric figure would be suitable for describing the mixing of *two* quantities? the mixing of *four* quantities? Give the details of your models.

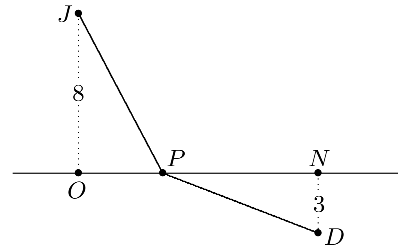
706 -1. In triangle RGB, point X divides RG according to $RX:XG = 3:5$, and point Y divides GB according to $GY:YB = 2:7$. Let C be the intersection of BX and RY.

(a) Find a ratio of whole numbers that is equal to the area ratio $CGB:CBR$.
(b) Find a ratio of whole numbers that is equal to the area ratio $CBR:CRG$.
(c) Find a ratio of whole numbers that is equal to the area ratio $CGB:CRG$.
(d) Find whole numbers m, n, and p so that $CGB:CBR:CRG = m:n:p$.
(e) The line GC cuts the side BR into two segments. What is the ratio of their lengths?

742 -1. A regular dodecagon can be dissected into regular polygons (which do not all have the same number of sides). Use this dissection (but *not* a calculator) to find the area of the dodecagon, assuming that its edges are all 8 cm long.

754 -1. In triangle RGB, point X divides side RG according to $RX:XG = m:n$, and point Y divides side GB according to $GY:YB = p:q$. Let C be the intersection of segments BX and RY. Find the area ratios
(a) $CGB:CBR$ **(b)** $CBR:CRG$ **(c)** $CGB:CRG$ **(d)** $CGB:CBR:CRG$
(e) Find the ratio into which the line GC divides the side BR.

805 -1. Jamie is at the point $J = (0, 8)$ offshore, needing to reach the destination $D = (12, -3)$ on land as quickly as possible. The lake shore is the x-axis. Jamie is in a boat that moves at 10 uph, with a motor bike on board that will move 20 uph once the boat reaches land. Find the landing point $P = (x, 0)$ that minimizes the total travel time from J to D. Assume that the trip from P to D is along a straight line.



805 -2. (Continuation) Let $O = (0, 0)$ and $N = (12, 0)$. Calculate the sine of angle PJO and the sine of angle PDN. These two sine values, together with the two given speeds, fit a simple relationship: the ratio of speeds is equal to the ratio of the sines of the angles. Try to predict what you would find if the boat's speed were increased to 15 uph. To validate your prediction, re-solve the preceding problem using the new speed. Write a general statement of this principle, known as *Snell's Law*, or the *Law of Refraction*.

811 -1. One stick is 3 ft long and another is 6 ft long. You break the longer stick into sections.
(a) If the sections are 2 ft and 4 ft long, will the sticks form a triangle?
(b) If the sections are 1 ft and 5 ft long, will the sticks form a triangle?
(c) If you break the longer stick at an arbitrary point, what is the probability that they form a triangle?

811 -2. Points D and E are marked on segments AB and BC, respectively. When segments CD and AE are drawn, they intersect at point T inside triangle ABC. It is found that segment AT is twice as long as segment TE, and that segment CT is twice as long as segment TD. Must T be the centroid of triangle ABC?

812 -1. Sam and Kirby were out in their rowboat one day, when Kirby spied a nearby water lily. Knowing that Sam liked a mathematical challenge, Kirby announced that, with the help of the plant, it was possible to calculate the depth of the water under the boat. While Sam held the top of the plant, which remained rooted to the lake bottom during the entire process, Kirby gently rowed the boat five feet. This forced Sam's hand to the water surface. When pulled taut, the top of the plant was originally 10 inches above the water surface. Use this information to calculate the depth of the water under the boat.

813 -1. Two sticks have length a and b with $a > b$. You break the longer stick at a random point. What is the probability that the resulting three sticks form a triangle?

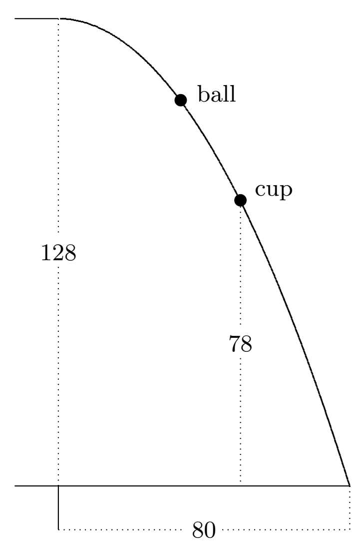
Math 2 Appendix

813-2. Trapezoid $ABCD$ has parallel sides AB and CD, of lengths a and b respectively. Diagonals AC and BD intersect at E. Draw the line through E that is parallel to AB and CD, and let P and Q be its intersections with AD and BC respectively.
(a) Prove that E is the midpoint of PQ.
(b) Show that $PQ = \dfrac{1}{\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}\right)}$. PQ is known as the *harmonic mean* of a and b.

813-3. You have seen that the midline of a trapezoid does *not* divide the trapezoid into two similar trapezoids. Is it possible that a *different* line (parallel to the midline) could divide the trapezoid into two similar trapezoids?

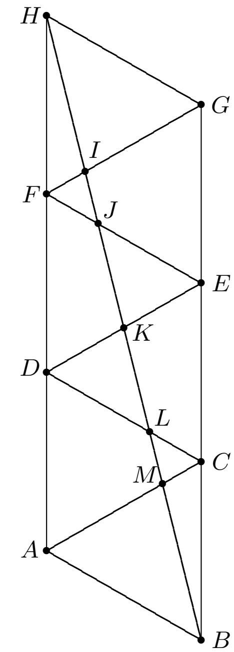
816-1. Recall that the lengths of the sides of triangle ABC are often abbreviated by writing $a = BC$, $b = CA$, and $c = AB$. Sketch triangle ABC where angle BCA is right and mark F as the foot of the perpendicular drawn from C to the hypotenuse AB. In terms of a, b, and c, express the lengths of FA, FB, and FC. The equation $c = FA + FB$ can be used to check your work.

820-1. After rolling off the end of a ramp, a ball follows a curved trajectory to the floor. To test a theory that says that the trajectory can be described by an equation $y = h - ax^2$, Sasha makes some measurements. The end of the ramp is 128 cm above the floor, and the ball lands 80 cm downrange, as shown in the figure. In order to catch the ball in mid-flight with a cup that is 78 cm above the floor, where should Sasha place the cup?



820-2. It is well known that $\dfrac{a}{b}+\dfrac{c}{d}$ is *not* equivalent to $\dfrac{a+c}{b+d}$. Suppose that a, b, c, and d are all positive. Making use of the vectors $[b,a]$ and $[d,c]$, show that $\dfrac{a+c}{b+d}$ is in fact *between* the numbers $\dfrac{a}{b}$ and $\dfrac{c}{d}$, while $\dfrac{a}{b}+\dfrac{c}{d}$ is not.

820-3. Let $A = (0,0)$, $B = (120,160)$, and $C = (-75,225)$, and let the altitudes of triangle ABC be segments AD, BE, and CF, where D, E, and F are on the sides of the triangle.
(a) Show that $(AF)(BD)(CE) = (FB)(DC)(EA)$.
(b) Show that this equation is in fact valid for *any* acute triangle ABC. (*Hint*: One way to proceed is to divide both sides of the proposed equation by $(AB)(BC)(CA)$.)



820 -4. The figure at right is built by joining six equilateral triangles ABC, ACD, CDE, DEF, EFG, and FGH, all of whose edges are 1 unit long. It is given that $HIJKLMB$ is straight.

(a) There are five triangles in the figure that are similar to CMB. List them, making sure that you match corresponding vertices.

(b) Find the lengths of CM and EK.

(c) List the five triangles that are similar to AMB.

(d) Find the lengths of CL, HI, IJ, and JK.