Geometer's Sketchpad Activities

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# Geometer's Sketchpad Introductory Activities

The series of computer activities over the next several pages are designed to help a raw beginner progress independently to the point of being familiar with the basics, and feeling reasonably comfortable using this extremely powerful yet user friendly geometry software. If possible, try and work through these activities with a partner. One person is the primary reader, while the other is the mouse/keyboard operator. Change roles every 20 minutes or at a convenient breaking point.

Activity I - Getting Started

1. After starting the program, the window you see in front of you consists of a Menu Bar, a sketch area which starts out blank, and a Toolbox along the left side of the sketch area. (See diagram to the right.) The default title of the sketch is Untitled1.gsp visible on the title bar. Click in the top right corner of both sizing boxes to increase both the Sketchpad window and the individual sketch window to full size.
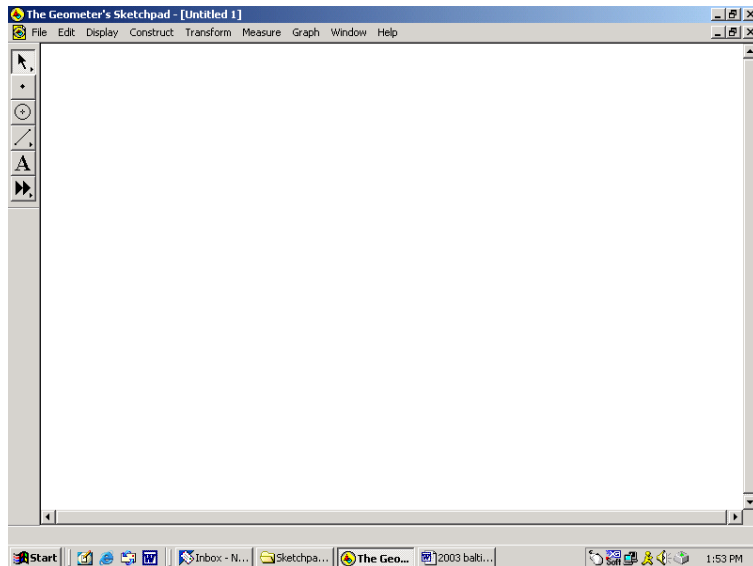
2. To start our first sketch, click on the Point Tool and move the cursor into the sketch window. Notice the cursor icon is now an arrow with a point at the end of it. Click the mouse's left button to construct a point, which should automatically be labeled A. If it is not labeled, that means that the automatic label feature for points is not activated. To activate it, click **Edit/Preferences**. (This is the notation that will be used to indicate first choose the Edit menu, and then choose Preferences from the submenu choices.) The dialogue box that opens up has three tabs for Units, Color, and Text. Choose the Text tab, and then activate the autoshow label feature. As in all dialogue boxes in Sketchpad, click **OK** or hit the *Enter* key on the keyboard to exit. Construct three other points by clicking the left mouse button in the sketch window. They will automatically be labeled in alphabetical order.

3. Let's assume the last point constructed was a mistake. Anytime you do something unintended or wrong, simply click **Edit/Undo** to erase the mistake. Try it, so as to eliminate your last point. Notice that the latest point constructed was always highlighted compared to the others. This is to indicate that this point is "selected". Selected objects are very important in GSP, (abbreviation for Geometer's Sketchpad), because they indicate to the program what objects you are using for constructions, or measurement, or transformations. In order to select or deselect objects, you need the Selection Tool**,** (the arrow)**,** from the top of the tool box. Get the select tool by clicking on the arrow icon, and then point the arrow at point A and click. Point A becomes selected. Now point at B and click. Point B also becomes selected.To deselect everything, click with the arrow on empty space.

4. Select all three points, and then click **Construct**/**Segment**. The triangle is constructed, and each segment is highlighted. This highlighting of the segments means they are all currently selected, because they were just constructed. Click in open space and the segments become deselected. To select them again, put the Selection Tool on each segment, and click. To deselect any individual object that is selected, put the Selection Tool on the object and click. See sample sketch to the right. To select segments, point arrow at segment and click, to select points, point the arrow at the point, (not the label), and click.
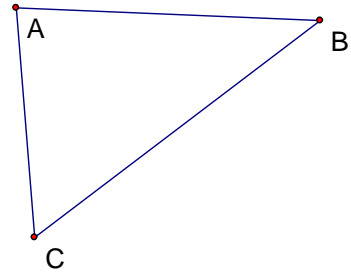
5. While all three segments are selected as shown to the right, click **Edit**/**Cut.** The three segments are deleted from the sketch. If that was a mistake, you can get them back by clicking **Edit**/**Undo Delete Segments**, and they are back, and still selected. Click **File**/**Close**/**Don't Save** to close the sketch window.

6. Start a new sketch by clicking **File**/**New Sketch.** A blank sketch window opens up entitled *Untitled02.gsp*. Click on the sizing box to make it full size. Select the Straight Edge Tool from the toolbox. Click, drag a few inches, and then release. A line segment AB is constructed. Now, place the cursor on point B, click, drag, and release to construct segment BC. To complete the triangle, place the cursor on either point A or C, click and drag over to the other point, and then release. If you are reasonably close to an already existing point, GSP assumes you wanted to hook up to that existing point and thus will not construct a new end point for the line segment.

7. Go back to the toolbox and get the Selection Tool. Select all three segments of the triangle. Click **Measure**/**Length.** The lengths of all three segments are displayed. The units and precision may be changed by clicking on **Edit**/**Preferences**, and making appropriate changes in the preferences dialogue box. Experiment if you like. To measure an angle, select the three points that name the angle, and click **Measure**/**Angle**. Measure angles *ABC* and *BAC*.

8. WARNING: If an option in a menu is grayed out, then you have either selected too few or too many objects. For example, selecting only two points will not be enough to measure an angle.

9. Using the Selection Tool, click and drag on any of the vertices or edges of the triangle and observe. This dynamic capability of GSP is what makes it so powerful. Notice, that all measurements change appropriately when the diagram changes. Also, it should be noted that the measurements themselves can be dragged around the window to any location that is desired. Always use the Selection Tool in any dragging operation. Move the measurements near the object they are measuring.

10. To sum up the three side lengths, click **Measure**/**Calculate...,** and a large calculator appears. You may move the calculator around by clicking and dragging it at the top. Move it so you can see the three side length measurements. Click on one of the side length measurements, and notice it appears in the calculator window. Enter a plus sign from the keyboard, or click on the plus sign on the calculator, and then click on a second side length measurement. When the sum is completely entered, click **OK**, or hit the *Enter* key. The sum of the three sides is now displayed.

11. Select the three vertices of the triangle, and click **Construct/Triangle Interior.** The triangle's interior becomes shaded with slanted lines running through it. The lines indicate that the interior is selected. If you click outside the triangle, the interior is deselected. Select the interior again by simply clicking inside the triangle, and click **Measure/Area.** Select the interior again, and choose **Measure/Perimeter**. Notice that area is in square units, and that the perimeter measurement agrees with the earlier calculation of adding up the three side lengths.

12. Select the polygon interior, hold the shift key down, and choose **Display**/**Color**. Change the color to one of your choice from the available palette. The reason for the shift key, was to prevent this new color from becoming the new default color for all future constructions. Hide the polygon interior by selecting **Display/HideTriangle**. Select the three sides, hold the shift key down, and click **Display**/**Line Width**/**Thick**. There are three line styles available, thin, thick, and dashed. Notice they are not the same color as the polygon interior. Practice by changing the color of the 3 sides without changing the default color. Click **File**/**Close**/**Don't Save** when finished.

Activity II - More constructions and using the Text Tool.

1. Start a new sketch, (**File**/**New Sketch**), and construct a random triangle ABC using any method you want. Construct the midpoint D of side AB by selecting the side and clicking on **Construct**/**Midpoint.** Construct segment CD. Now, in order to construct the altitude from B, select B and side AC, and choose **Construct**/**Perpendicular Line**. Your sketch should look like the one at right.
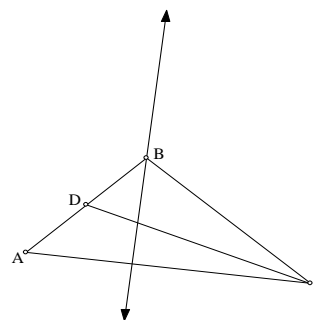
2. To construct a point of intersection, GSP has a couple of different methods to choose from. Method 1: Select both objects that are intersecting, and choose **Construct**/**Intersection**. Method 2: Simply point the Selection Tool at the intersection, and click. The Point Tool cursor can be used the same way as the Selection Tool to construct a point of intersection. Use each method once to construct the intersections of the line through B with CD and AC. Drag on the figure, and of course the intersections move dynamically with everything else.

3. What if we wanted to see the intersection points we just constructed, but did not want to see the lines anymore. Select the line through B as well as segment CD, and click **Display/Hide Straight Objects.** The lines are hidden from view only. The points will still behave as they did before. Hiding objects is very important, and often used to keep diagrams from getting too cluttered. To see them at any time, simply click **Display**/**Show all Hidden.** Try it.

4. IMPORTANT: There is a big difference in GSP between hiding objects and cutting or clearing them using the Edit menu. Cutting or clearing an object also forces the deletion of all objects that depended on the one eliminated. Hiding simply removes the object from view. To illustrate, this time select CD and click **Edit**/**Cut.** Notice that not only did CD disappear, but so did the intersection point that was on CD. Now the only way to get it back is to choose **Edit**/**Undo Deleted Objects.** Your sketch should be back to looking like the one to the right.
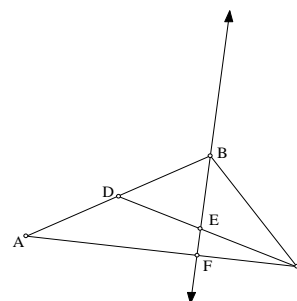
5. Sometimes it is important to move labels, change label names, hide label names, or even annotate a sketch. All of these are accomplished using the Text Tool, the tool indicated by the capital **A** in the tool box. . Select the Text Tool, and point the cursor at point A, not label A. (You know you are close enough to the point when the hand cursor turns dark.) Click the mouse and label A is hidden. Click again, and it returns. It is simply a toggle switch that turns on and off the labels. Try it on side AC, whose segment label is currently hidden because autoshow labels for segments is not activated. When the label appears, it will be a lower case letter, since GSP defaults to capital letters for points and lower case letters beginning with j for straight objects.

6. However, GSP also allows the user to change any label to any name they want. It even allows for a user to have the same label for different objects. This is not a good idea, and should be avoided. To practice changing label names, point the Text Tool cursor at the label B this time, rather than the point B. (You know you are close enough when the letter A appears on the hand.) Click and drag to move the label. Notice the label for an object will not be moved too far from the object itself. To change the label's name, double click the mouse button to open up a dialogue box that allows for a label change. You can also control font type and size for individual labels from this dialogue box. Practice by changing the three vertices from A,B,C to Alpha, Beta, and Charlie.

7. To annotate a diagram, also known as creating a caption, use the Text Tool to open up a text box on your sketch by clicking and dragging in open space. After the text box is opened, type in the text you want, and simply click outside the box when finished. Options for text size, color, and mathematical symbols are available in a text menu at the bottom of the sketch. The Selection Tool can be used to drag the caption anywhere in your sketch. To subsequently edit the text, use the Text Tool and click on the text and start typing. The caption will resize automatically. Create a caption similar to the one shown at right entitled Activity II. Notice the font has been enlarged, and is italicized. Click **File**/**Close**/**Don't Save.**

Activity II
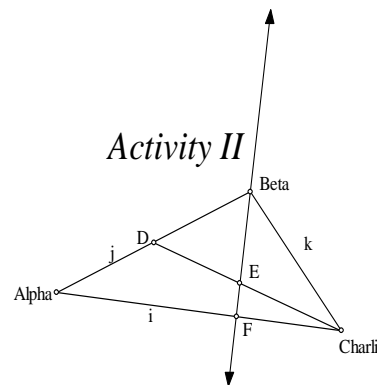
Note: GSP always uses default label names in alphabetic order for points and straight objects. Even if something is deleted or undone, that label name will not automaticaly be reused. To resequence default label names for points from a new starting point, select two or more points that are to be resequenced in the order you want them, choose **Display/Label Points,** and enter the beginning of the new sequence you would like to use.

Activity III - Coordinate Geometry and Transformations

1. Start a new sketch, and make sure the distance unit is **cm** by clicking on **Edit/Preferences**. To construct the triangle with vertices (6,1), (–5,4), and (3,–2), click **Graph/Plot Points,** and enter in the coordinates in the dialogue box, followed by Plot each time. Use the *Tab* key to move around any GSP dialogue box most easily, and click **Done** when finished. Choose **Construct/Segment** to construct the triangle. Click in open space to deselect. Drag on any of the points, and notice that everything moves together. This is because the points were constructed with specific coordinates.

2. Also notice, that the origin (0,0), and the unit point (1,0) are automatically constructed whenever the axes are used and points plotted. Plot the three points (15,20), (–12,–14), and (20,–10). It may seem that GSP did not plot the points, but you just can't see them because they are off the screen. Drag the unit point closer to origin, and watch what happens. The scale is always controlled by unit point . The new points will eventually come into view as the unit length becomes smaller. Click and drag on the origin. Notice that you are allowed to move the origin to any convenient location in your sketch window. Select the six triangle vertices, and click **Measure/Coordinates**. Click in open space to deselect. When you drag on any of the six points, they will all still move together, and the coordinates of the points will not change.

3. Start a new sketch, (**File/New Sketch**), and click **Graph/Show Grid** after making the window full size. Use the Point Tool to construct three points anywhere on the grid. Select all three points and click **Measure/Coordinates**. The coordinates are displayed as in previous example. However, unlike in the previous example, these three points will move freely when dragged. Try it, and notice the coordinates for any point moved will change appropriately. Remember, we did not close the previous sketch before starting this one. It is still an open sketch, and just not visible. Think of the open sketches as layered on top of each other. Click on **Window** , and at the bottom of the menu is a listing of all the open sketches with a check mark next to the one that is currently active. If you want to go back to any previous sketch that is open, simply click on its name. Try it by moving back and forth a couple of times between open windows.

4. Start a new sketch, (**File/New Sketch**), and click **Graph/Plot Points** after making the window full size. Plot the point (5,0), which should be labeled A. Select the origin first, and then A. Choose **Construct/Circle by Center and Point.** The circle with center at the origin and a radius of 5 cm is now constructed. This is one way to construct circles with a fixed radius that won't change. Note that it was important to select the center first. Hide the axis and let's try another way to construct a fixed radius circle.

5. Construct a random point using the <u>Point Tool</u>. This should be labeled B. With point B selected, click **Transform/Translate**. The translation dialogue box opens up. Click in the box adjacent to the word Rectangular. Enter a 5 for Horizontal, 0 for Vertical, and click **Translate**. Remember, use the *Tab* key to move around any GSP dialogue box most easily. A new point labeled B' is constructed that is 5 cm to the right of B. Drag on B, and B' will move also. Select B first, then B', and click **Construct/Circle by Center and Point.** This is another way to construct a circle with a fixed radius.

6. A third method to construct a circle is the following. Use the <u>Segment Tool</u> to construct a random segment which should have endpoints labeled C and D. Use the Point Tool to construct a point E. Select E first and then the segment and click **Construct/Circle by Center and Radius**. The circle's radius will stay the length of segment CD.

7. To construct a circle with a varying radius, simply select the Circle Tool from the Tool Box, and click and drag in your sketch window. The circle constructed has a labeled center point and one other point on the circle constructed as well. Use the Selection Tool to drag on the center, the point on the circle, and the circle itself, in order to become familiar with how the construction behaves. Click **File**/**Close**/**Don't Save.**

8. Start a new sketch, (**File**/**New Sketch**), and click **Graph**/**Plot Points** after making the window full size. Construct the triangle C(3,1), D(5,2), E(6,−5) by plotting the points and constructing the segments. Click **File**/**Save**, and enter in *Tr1* as the filename. We want to reflect the triangle about the line y = 2x, and give the coordinates of the vertices of the image triangle. To get the line y = 2x, plot the point (1,2) and use the **Construct** menu construct the Line through (0,0) and (1,2). Notice that GSP gives you three straight construction choices from which to choose when two points are selected. This is also true with the Straight Edge Tool from the tool box. Let's try another method for constructing the line using a tool. Undo the line you just constructed, and then click and hold on the Straight Edge Tool in the tool box. Notice it is a three in one tool that allows you to choose among segment, line, or ray. Choose the line icon, and then click and drag from the origin to the point (1,2). Make sure you release the mouse when you are on the point (1,2). We will learn another method for constructing y = 2x later in these activities.

9. While the line is still selected, click **Measure**/**Equation.** The equation of the line is displayed. This verifies that indeed you have constructed the correct line. (Note: GSP can display equations of lines and circles only.) While the line is still selected, click **Transform**/**Mark Mirror**. Two squares on the line will temporarily "explode". This is the indication, that the line has been marked as a mirror. Using the Selection Tool, click and drag open a box that encompasses the three vertices and edges of the triangle. When you let go, everything within the box will be selected. This *selection box method* is an easier way to select a large group of objects rather than doing it one click at a time. Notice you also have selected the x-axis which you do not want to reflect. Deselect the x-axis and then click **Transform**/**Reflect**. The image triangle appears automatically labeled with primes ('), added on to the pre-image triangles labels. Select only the 3 vertices of the image triangle, and choose **Measure**/**Coordinates**. Move the coordinates in close proximity to the actual point, and your sketch should look like the one at right. Click **File**/**Close**/**Don't Save.**
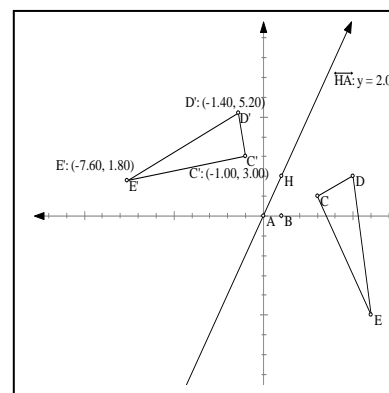
10. Click **File**/**Open**, and enter *Tr1* as the file name, or choose it from the list given in the dialogue box. Click **OK**. The goal here is to rotate the triangle CDE ninety degrees counterclockwise about the center point (1,−1), and to display the coordinates of the vertices of the image triangle. Click **Graph**/**Plot Points** to plot the point (1,−1). While it is still selected, click **Transform**/**Mark Center**. Notice how the point "explodes". This indicates it has been marked as a center point. Select the vertices and edges of the triangle and choose **Transform**/**Rotate**. Enter 90 for the angle. Positive angles rotate counterclockwise, negative angles rotate clockwise. After the image triangle appears, display the coordinates of the vertices by selecting them and clicking **Measure**/**Coordinates**. When finished, the sketch should look like the one at right. Click **File**/**Close**/**Don't Save.**
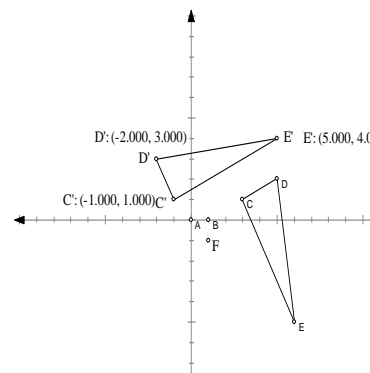
Activity IV- Vectors and Dilations

1.   Start a new sketch, (**File**/**New Sketch**), and use the Segment Tool to construct a random segment AB. Now, use the Point Tool to put a point C anywhere off the line.

2.   Use the Selection Tool to select point A first, and then B. Click **Transform**/**Mark Vector.** Notice how the segment was highlighted temporarily going from A to B. Now select point C and click **Transform**/**Translate**. This time, when the translation dialogue box opens up, the box next to By Marked Vector is activated. Click **Translate,** and the point C' is constructed so that CC' are related exactly the same to each other as AB. Drag A or B, and see how C' moves to maintain the relationship. Use the Segment Tool to construct the quadrilateral ABC'C as shown to the right. ABC'C is a parallelogram where both pairs of opposite sides are parallel. Select all four sides, and click **Measure**/**Slope** to verify that this is the case. Click **File**/**Close**/**Don't Save.**
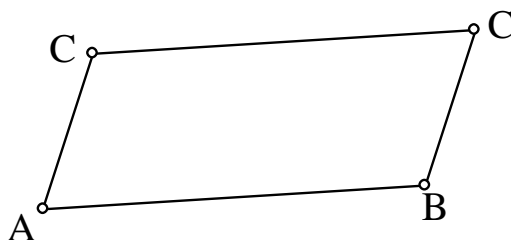
3.   Start a new sketch, and construct a random triangle ABC, and a point D outside the triangle. Double-click on point D, and notice it "explode" the same way as when we chose **Transform**/**Mark Center** for a point in an earlier activity. Double-clicking is a short cut for marking a point as a center. Select the entire triangle. Try using the *selection box method* described in paragraph #9 on page 7. After selecting the entire triangle, click **Transform**/**Dilate.** A dialogue box requires a ratio for the scale factor. (The default is usually 1(New)/ 2(Old).) Enter these values if they are not already there. Click **Dilate.** The new triangle formed, A'B'C', is exactly half the size as the original, exactly the same shape, and half as close to center point D as the original. Drag D and observe.

4.   Drag on the triangle and observe. Select all of triangle A'B'C' and repeat the dilation. There is no need to mark D as the center point again. Have fun dragging on the various objects in the sketch. Practice coloring in the triangles for a nice visual effect. Measure areas and look at their ratios as a reminder of how area ratios and linear measurement ratios compare.

Activity V – Function Plotting

1. GSP versions 4.0 and above have a function plotting capability that did not exist on earlier versions. Start a new sketch, and select **New Function** from the **Graph** menu. The New Function dialogue box opens up that also is similar to the one displayed when **Calculator** is chosen from the **Measure** menu. You will only type in the expression that will become the right hand side of the function. By clicking on the appropriate buttons on the calculator, put x^2-2 on the display, and then click OK. Notice the function $f(x) = x^2 - 2$ is displayed in the sketch window. While it is still highlighted, select **Graph/Plot Function.** A coordinate system and the parabola will appear in your sketch.

2. Create another function graph by repeating the process above, and this time enter in the linear expression 2x+1 in the calculator after selecting **Graph/New Function**. This time the function is called $g(x)$. Plot this function, and change its color to contrast with the parabola. Although it is clear that these two functions intersect, GSP does not find the intersection points between functions the way it does for lines and circles. Even linear functions are **not** treated the same way as two lines constructed using the linear tool or the **Construct** menu and two points.

3. Editing a function is easy. Simply double-click on the function, (not the graph), with the <u>Selection Tool,</u> and the function appears in the Calculator window ready for editing. After editing and clicking OK, the function is changed, and the graph of that function is also changed automatically.

4. In creating new functions, GSP allows the user to access many of the standard functions that are located in the Functions drop down menu in the New Function dialogue box. Try creating and graphing a new function that involves one or more of these built-in functions. Perhaps something like $\sin(abs(x))$ would be fun to try.

5. GSP allows for x = f(y) format and r = f(θ) format for graphs. Try $x = y^2$ and $r = 2 + 2\sin(\theta)$, and your picture should look like the one at right.

6. Composite functions are also easily explored with GSP. After opening the New Function dialogue box, a user can click on any function already defined in the sketch window, and it will appear in the function window. Typing in expressions like $f(g(x))$ or $g(f(x))$ allows the user to define composites of previously defined functions f and g and explore their graphs. Make up your own examples and try it.
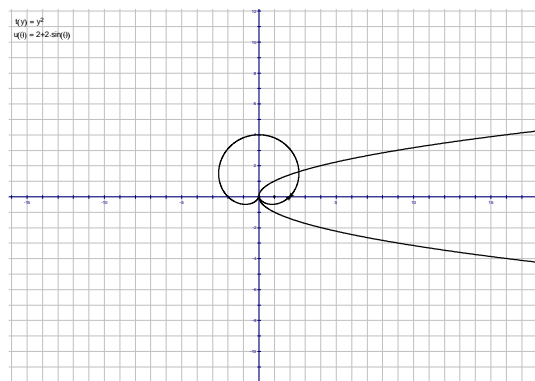
Activity VI – Using Custom Tools

1. The last tool in the tool box is the <u>Custom Tool</u> that enables users to create their own convenient tools for constructions that are repeated often. This is a an advanced feature of GSP, but can be effective and very useful. We will create a couple of simple tools in this activity so you will be encouraged to try more on your own. Start a new sketch.

2. Use the <u>Segment Tool</u> to draw a segment AB in the middle of your sketch window. We are going to construct a square using rotations. Change to the <u>Selection Tool</u> and double-click on the left end point of your segment, and notice how the point explodes. (A quick way to change to the <u>Selection Tool</u> from any other tool is to hit the escape key on the keyboard). This indicates the point has been marked as a center point for a transformation. Select the segment and the other endpoint, and then choose **Transform/Rotate.** Enter in 90 for the angle of rotation, and click on the rotate button. The segment and the point are rotated 90 degrees counterclockwise. To rotate clockwise, negative angle amounts are used. Now, mark this new point as the center, and rotate the adjacent segment and point 90 degrees as well. Your sketch should now look like the one at right.

3. Finish the square by constructing the last side. Now select just the vertices of the square and choose **Construct/Quadrilateral Interior**. While the interior is still selected, choose **Measure/Area.** Let's get the area measurement inside the square permanently even when the square is dragged around. First, construct the center point of the square using the intersection of the diagonals. Select and then hide the diagonals using the Display menu. Select the area measurement and the point inside the square and then hold the shift key down while clicking on the **EDIT** menu. Choose **Merge Text to Point**. Notice how the area moves right on top of the point in the middle of the square. Hide that middle point and the area measurement that is still outside the square. Now it is time to actually create the custom tool.

4. Click on the <u>Selection Tool</u>, choose **Edit/Select All**, and then click and hold on the Custom Tool button. Select Create New Tool from among the choices in the drop down menu. A dialogue box opens up. Type in the name *square maker* for this tool. Also, click on the box that says show script view. A new box will open up that can be enlarged by dragging on the lower right hand corner of the box. The script in this box details all the constructions that were used to make the square. Importantly, it also indicates that the needed elements to use this new Tool that will make a square are two points. Click OK.

5. Now it is time to try out this new tool. Click on the custom tool icon and then click and drag anywhere in the sketch window. GSP will automatically construct a square with one vertex at the first place you clicked, and the adjacent vertex at whatever location you click again. You can also use already existing points as your two needed points. Click and drag from a vertex of an exisiting square to the diagonally opposite vertex and then release. Another new square is created!

6. This new tool is now available to use in this sketch and in any other sketch as long as this document remains open. There is also an easy way of permanently making custom tools available on all future sketches. After creating a custom tool in a sketch and naming it, click on the **Save As** option under the **File** menu. and then save the sketch in the Tool Folder that is automatically created in the Sketchpad folder when the software was installed as a single user copy on your own stand alone computer. GSP looks in this Tool Folder whenever the program is first started, and makes the custom tools located there available to the user. Do this now, so the tool *square maker* will be available for future sketches.Unfortunately, this is not available if you are on a network computer.

Computer Lab #1 – Slope

1. Once you have opened the program Geometer's Sketchpad, click on menu **File/Open**, and choose file entitled ***Linear41.gsp***. When the sketch on the screen opens up, you should see a coordinate grid with a line constructed through the two points labeled C and D. The coordinates of C and D should be displayed in the top left hand corner of the sketch window along with the slope of the line. Your sketch should like the one displayed at right.

   Linear1

   C: (-1.00, 0.00)
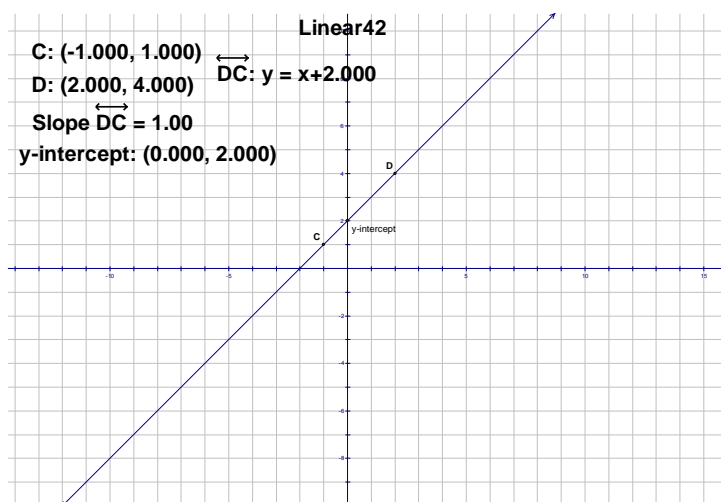   **D: (2.00, 3.00)**
   Slope DC = 1.00
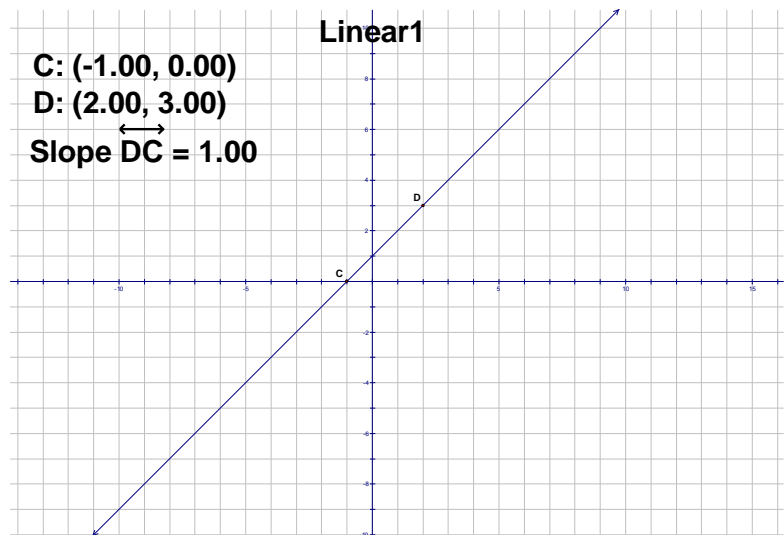
2. Using the arrow tool on the screen, which is controlled by the mouse, click on point C and drag it to the origin. Then drag point D to the location (1,1). The program is in a mode that only allows points C and D to land on points with integer coordinates. These points are known as *lattice points*, or *grid points*. Drag C and D to different locations and notice the change in the slope of the line.

   a) How would you describe lines that have positive slope?

   b) How would you describe lines that have negative slope?

   c) How would you describe lines that have zero slope?

   d) How would you describe lines that have undefined slope?

3. Drag on the line instead of either C or D and observe its behavior and the value of the slope. Summarize your observations below.

4. Move point C to the indicated location in the table below, and then find the coordinates for point D so that the slope of the line CD is the number in the left column.

Slope	Coordinates of C	Coordinates of D
2.000	(0,0)	
−3.000	(2,3)	
0.000	(-1,4)	
1.250	(2,-5)	
Undefined	(3,1)	
−8/3	(-1,-2)	
1.125	(-3,2)	
3.500	(4,0)	

Computer Lab #2 – Slope and Lattice Points

Once you have opened the program Geometer's Sketchpad, click on menu **File/Open**, and choose file entitled **Linear42.gsp**. When the sketch opens, you should see a coordinate grid with a line constructed through the two points labeled C and D. The coordinates of C and D should be displayed in the top left hand corner of the sketch window along with the slope of the line, the coordinates of the *y*-intercept of the line, and the equation of the line in the *Slope-Intercept* form of $y = mx + b$.

C: (-1.000, 1.000)
D: (2.000, 4.000)
$\overleftrightarrow{DC}$: y = x+2.000
Slope $\overleftrightarrow{DC}$ = 1.00
y-intercept: (0.000, 2.000)

Linear42

Points C and D completely determine the line. The program is in a mode that only allows points C and D to to be dragged to locations with integer coordinates. However, the *y*-intercept may end up at a non-integer location, but not through dragging on it. Only points C amd D are to be moved in the exercises that follow. Drag point C to the origin, and then move D to the appropriate location so as to produce the line whose equations are given below. Record the location of point D for each of these lines.

Equation of Line	Coordinates of C	Coordinates of D
$y = 4.000x$	(0,0)	
$y = -3.000x$	(0,0)	
$y = 1.500x$	(0,0)	
$y = 0.000$	(0,0)	
$y = -1.400x$	(0,0)	
$x = 0.000$	(0,0)	
$y = 2.750x$	(0,0)	
$y = -0.\overline{33}x$	(0,0)	
$y = 0.375x$	(0,0)	

Complete the chart below by filling in locations for both C and D so as to produce the line whose equation is given in the first column of the table.

Equation of Line	Coordinates of C	Coordinates of D
$y = 2.000x - 3.000$		
$y = -1.500x + 4.000$		
$y = 2.000$		
$y = 1.750x - 1.000$		
$x = 4.000$		
$y = -1.600x + 1.000$		
$y = 1.333x - 5.000$		

This exercise is exactly like the #3 above, except the y-intercepts are not at lattice points. This only means that neither C or D will be located on the *y*-axis. Find locations for C and D that will form the line given in the left column of the table below.

Equation of Line	Coordinates of C	Coordinates of D
$y = 1.500x - 2.500$		
$y = -0.600x + 1.800$		
$y = 2.750x + 0.500$		
$y = 1.333x - 2.667$		
Y=-0.375x+1.250		

Explain in the space below why there are an infinite number of lattice points on any one of the lines given above.

In the space below, explain or demonstrate why there are no lattice points on the line
$y = 1.200x + 1.500$

Computer Lab #3 – Slope-Intercept Sliders

Once you have opened the program
Geometer's Sketchpad, click on the
menu File/Open, and choose the file
entitled ***Linear43.gsp***. When the
sketch on the screen opens up, you
should see a sketch like the one on the
right. There is a coordinate axis with
two "*sliders*" you will use to change
the value of the *parameters m* and *b* in
the equation $y = mx + b$. These two
parameters control the line in red that
is displayed on the screen. Also
displayed are the values for *m*, *b*, and
the *y*-intercept of the red line.
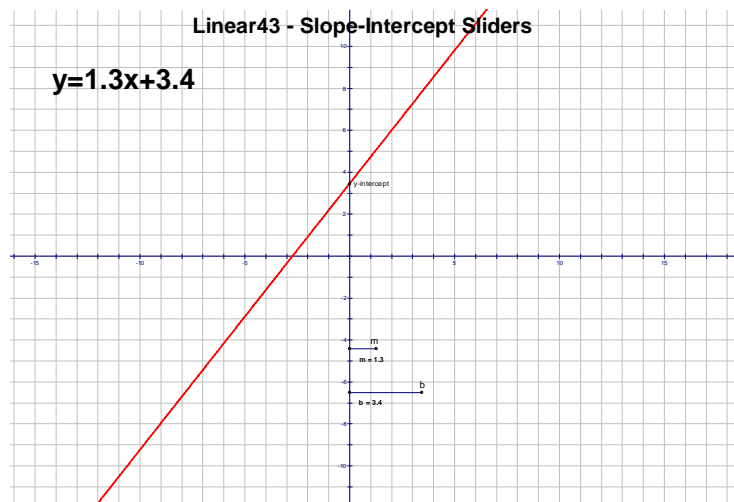
Linear43 - Slope-Intercept Sliders

$y=1.3x+3.4$

y-intercept

m
m = 1.3

b
b = 3.4

General Instructions: For the purpose of this worksheet, the accuracy for these parameters have been
purposefully chosen to be one decimal place. It is important for you to realize this limitation on the
accuracy, so that for all problems you should check your answers by hand calculations in order to
make any necessary small adjustments. Technology is a marvelous tool, but always know the
limitations of the technology you are working with. You will be dragging on the points along the
sliders that are labeled *m* and *b*. This will change the values of these parameters and thus change the
red line in the sketch.

Click and drag on the point labeled *m*.
Describe below what happens to the red line when *m* becomes increasingly positive.

What happens to the line if *m* equals zero?

Describe below what happens to the red line when *m* becomes increasingly negative.

Summarize the role *m* plays in the equation $y = mx + b$.

Drag the slider *m* so that its value is back to 1.0 Now drag slider point *b* and observe.
Describe below what happens to the red line as *b* changes. Focus your attention on the y-intercept.

Drag slider point *b* so that its value is 3.0. Now move slider point *m* back and forth. Move the slider point *b* so that its value is −2.0. Move slider point *m* back and forth again. Describe below what behavior in the red line you observe.

Summarize the role *b* plays in the equation $y = mx + b$.

Write the equation in slope-intercept form for each of the lines described below. As a check, drag the *m* and *b* slider points to the appropriate values so that the line described is drawn on the screen. Remember, the accuracy of the slider values is only to one decimal place. On some problems you will need to do some paper and pencil calculations as a check to make sure you have the correct equation of the line.

Slope is 2.0, and the *y*-intercept is (0,−3).

Slope is −1.5 and the *y*-intercept is (0,4).

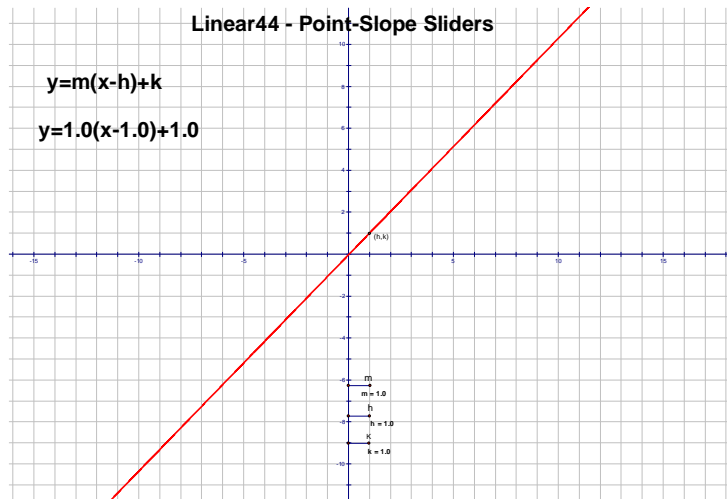Slope is 3.0 and the *x*-intercept is (−2,0).

Slope is −0.4 and contains the point (−2,4).

Contains the points (3,5) and (−1,3)

Computer Lab #4 – Point-Slope Form Sliders

1. Once you have opened the program Geometer's Sketchpad, click on menu **File/Open**, and choose file entitled **Linear44**. When the sketch on the screen opens up, you should see a sketch like the one on the right. There is a coordinate axis with three "*sliders*" you will use to change the value of the parameters *m*, *h*, and *k*. These three parameters control the line in red that is displayed on the screen. Also displayed in the upper left hand portion of the sketch is the equation in point-slope form.

Linear44 - Point-Slope Sliders

y=m(x-h)+k

y=1.0(x-1.0)+1.0

General Instructions: For the purpose of this worksheet, the accuracy for these parameters has been purposefully chosen to be one decimal place. It is important for you to realize this limitation on the accuracy, so that for certain problems you will check your answers by hand calculations in order to make any necessary small adjustments. Technology is a marvelous tool, but always know the limitations of the technology you are working with. You will be dragging on the points along the sliders that are labeled *m*, *h*, and *k*. This will change the values of these parameters and thus change the red line in the sketch whose equation is determined by these parameters in the form $y = m(x - h) + k$.

2. Click and drag on the point labeled *m*.

a) Describe below what happens to the red line when *m* becomes increasingly positive.

b) What happens to the line if *m* equals zero?

c) Describe below what happens to the red line when *m* becomes increasingly negative.

d) What one point remains unchanged when m changes?

Summarize the role m plays in the equation $y = m(x - h) + k$ **.**

3. Drag slider point *m* so that it's value is equal to 1.0. Now drag slider point *h* and observe.

a) Describe below what happens to the red line as *h* changes. Focus your attention on the point (*h,k*) that is on the line.

b) Drag slider point *h* so that its value is 3. Now move slider point *m* back and forth. Move slider point *h* so that its value is −2. Move slider point *m* back and forth again. Describe below what behavior in the red line you observe.

c) Summarize the role *h* plays in the equation $y = m(x - h) + k$.

4. Drag slider points *m* and *h* so their values are one. Now drag slider point *k* and observe.

a) Describe below what happens to the red line as *k* changes. Focus your attention on the point (*h,k*) that is on the line.

b) Drag slider point *k* so that its value is 3. Now move slider point *m* back and forth. Move slider point *k* so that its value is −2. Move slider point *m* back and forth again. Describe below what behavior in the red line you observe.

Summarize the role *k* plays in the equation $y = m(x - h) + k$.

5. Write the equation of the line in *pt-slope form* for each of the lines described below. As a check, drag the *m*, *h*, and *k* slider points to the appropriate values so that the line described is drawn on the screen. Remember, the accuracy of the slider values is only to one decimal place. On some problems you will need to do some paper and pencil calculations as a check to make sure you have the correct equation of the line.

Slope is 2 and contains the point (1,3).

Slope is 2 and contains the point (−2,1).

Slope is −1 and contains the point (−2,1).

Parallel to the line $y = 3(x-2)+4$, and goes through the origin.

Slope is 4/5 and the x-intercept is (2, 0).

Contains the two points (2,3) and (−1,4).

Contains the two points (−3,5) and (4,5)

Explain below why the line through the points (2,3) and (2,-2) cannot be constructed using this slider program, implying that its equation cannot be written in *pt-slope form.*

Computer Lab #4A – Standard Form of Linear Functions

1. Once you have opened the program Geometer's Sketchpad, click on File/Open and choose file entitled linearstd.gsp. There is a coordinate axis with three "*sliders*" you will use to change the values of the parameters *A*, *B*, and *C*. These three parameter values control the line in red that is displayed on the screen. Also displayed in the upper left hand portion of the sketch is the equation of the line in standard form $Ax + By = C$, the coordinates of the *x* and *y* intercepts, and the slope of the line.

   General Instructions: The values for *A*, *B*, and *C* are limited to integers. When you need to display a line by changing the parameters, it is necessary that the line be written so that *A*, *B*, and *C* are integers. You will be dragging on the points along the sliders that are labeled *A*, *B*, and *C*. This will change the values of these parameters and thus change the red line in the sketch whose equation is determined by these parameters.

2. Click and drag on the slider point labeled *A*.

a) Describe below what happens to the line when *A* becomes increasingly positive.

b) What happens to the line if *A* equals zero?

c) Describe below what happens to the line when *A* becomes increasingly negative.

d) Summarize the role *A* plays in the equation $Ax + By = C$. Try and be as specific as possible with respect to the slope and the *x*-intercept.

3. **Click and drag on the slider point labeled B.**

a) Describe below what happens to the line when B becomes increasingly positive.

b) What happens to the line if B equals zero?

c) Describe below what happens to the line when B becomes increasingly negative.

d) Summarize the role B plays in the equation $Ax + By = C$. Try and be as specific as possible with respect to the slope and the y-intercept.

4. **Click and drag on the slider point labeled C.**

a) Describe below what happens to the line when C becomes increasingly positive.

b) What happens to the line if C equals zero?

c) Describe below what happens to the red line when C becomes increasingly negative.

d) Summarize the role C plays in the equation $Ax + By = C$. Try and be as specific as possible with respect to the slope and the axis intercepts.

5. Write the equation of the line in *standard form* for each of the lines described below. As a check, drag the *A*, *B*, and *C* slider points to the appropriate values so that the line described is drawn on the screen. Remember, the *A*, *B*, and *C* slider values must be integers, preferably relatively prime. Do the paper and pencil calculations **first** , and then use the program as a check to make sure you have the correct equation of the line. In some cases, your equation might be easiest to initially write in another form, and then transformed into standard form.

a) Parallel to $3x+2y=4$ and contains the point (1,2).

b) Perpendicular to the *y*-axis and containing the point (3,4).

c) Parallel to the *y*-axis and containing the point (−1,5).

d) Slope is −2 and contains the point (−2,1).

e) *x*-int is (3,0) and *y*-int is (0,2)

f) Contains the two points (3,2) and (−1,4).

g) Perpendicular to the vector [−2,5] and passing through the origin.

Computer Lab #4B – Solving Simultaneous Equations Using Linear Combinations

The following type of problem in algebra is very common:

An 8-pound mixture of M&M's and raisins costs $18. If a lb. of M&M's costs $3, and a lb. of raisins costs $2, then how many pounds of each type are in the mixture?

Problems like this are easily modeled with a system of linear equations $\begin{cases} x+y=8 \\ 2x+3y=18 \end{cases}$

Where x = lbs of raisins, and y = lbs of M&M's. To solve such a system, one method that is commonly used is called the method of **"linear combination"**. A linear combination of two equations L1 and L2 is the sum of a constant m times L1 plus a constant n times L2. Thus a new line that has the equation $m*L1 + n*L2$ is created. Verify that equations below are all examples of linear combinations for the system from the M&M and raisin example above. Create at least 3 other examples of your own by choosing integer m and n values of your choice.

$m*$ $x+y=8$	$n*$ $2x+3y=18$	Linear Combination
1	1	$3x+4y=26$
2	1	$4x+5y=34$
1	−1	$-x-2y=-10$
2	−1	$0x-y=-2$
3	−1	$x+0y=6$

The first three examples of linear combinations in the table above do not appear to help us solve this problem. Why? However, the 4th and 5th example in the table are a different story. Since one of the variables is eliminated in the linear combination because its coefficient becomes zero, this enables us to solve the equation for the one variable that remains. In the 4th row example, the solution yields $y = 2$. This would allow a simple substitution of that value for y back into either of the original equations to determine that $x = 6$. Similarly, the 5th row example yields the answer $x = 6$ first, and then substitution back into an original equation would reveal that $y = 2$.

General Instructions: Open the file *linear combination* in Geometer's Sketchpad and a sketch should appear like the one at right. There are three lines that are in the sketch. Line1 and Line 2 are blue and red respectively, and are controlled by the blue and red sliders. Both lines are in standard form. The blue line ($Ax+By=C$) is controlled by the blue A,B, and C sliders, while the red line ($Dx+Ey=F$) is controlled by the red D,E, and F sliders. The slider parameters

can ONLY take on integer values. If you have previously worked with the *linearstandard* sketch, then you should be familiar with how these sliders control the lines. If not, spend a few moments moving the sliders back and forth and noticing the effect on the lines.
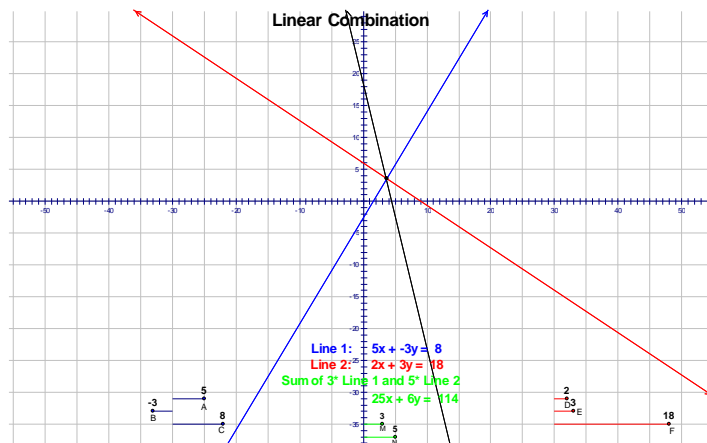
1. The green line is the linear combination line of the blue and red lines. Integer sliders also control this green line, but in this case, the green sliders consist of the multiples m and n for lines Line1 (blue) and Line2 (red) respectively. By changing the slider values m and n, a linear combination line in green is created. Go back to the table on the first page of this lab and use the blue and red sliders to graph the original system $\begin{cases} x+y=8 \\ 2x+3y=18 \end{cases}$. Then, change the m and n sliders appropriately to display the green linear combination line for each of the examples in the table and for the 3 other examples that you created yourself. Something very interesting and important is always true about the green linear combination line no matter what the values of m and n. In the space below, write what you observe, and try and explain why?

2. You should also observe something very important and interesting about the green linear combination line in the two special cases (4th and 5th examples in the table) that eliminated one of the variables and thus enabled an easy solution to the problem. Write your observations in the space below.

3. The method of linear combination for a system of the standard form $\begin{cases} Ax+By=C \\ Dx+Ey=F \end{cases}$ is very easy, especially when all the coefficients are integers. The basic idea is to determine values of m and n that will produce a linear combination equation that eliminates either the variables x or y by making the coefficient zero. Verify algebraically that in the system $\begin{cases} 3x-4y=10 \\ 2x+5y=-4 \end{cases}$, values of $m=5$ and $n=4$ would eliminate the y variable from the linear combination line, while values of $m=2$ and $n=-3$ would eliminate the x variable. Go to sketchpad and model the system above with the red and blue sliders, and then verify with sketchpad that the pairs of m and n values above do indeed produce a desired result. Are these the only pairs of relatively prime m and n values that will work, or can you come up with other examples. Try to do so, and then use the computer as a check.

4. What happens if the system is something like $\begin{cases} 1.2x - 2.5y = 2 \\ 3.2x + 5.6y = 8 \end{cases}$, or $\begin{cases} y = 3x - 2 \\ \dfrac{2}{3}x + 5y = 7 \end{cases}$? To use linear

combination and the program as a checking device, first put the equations in standard form as shown in #3 above by multiplying through by the appropriate constants to eliminate decimals and fractions. Doing so would change the systems above to $\begin{cases} 12x - 25y = 20 \\ 4x + 7y = 10 \end{cases}$ and $\begin{cases} -3x + y = -2 \\ 2x + 15y = 21 \end{cases}$. Verify this yourself. As a

challenge, use the space below to try and solve both of these systems algebraically and then check your solutions with sketchpad. Both solutions consist of numbers that are "not nice".

5. For each of the following systems, first solve them algebraically, and then go to sketchpad to check your answers and to see a visual representation of the system and the linear combination.

a) $\begin{cases} 5x + 3y = 8 \\ 4x + 3y = -2 \end{cases}$

b) $\begin{cases} 4x + 3y = 20 \\ 3x - 2y = -5 \end{cases}$

c) $\begin{cases} 2x + 7y = 14 \\ y = 2x + 1 \end{cases}$

d) $\begin{cases} 3x - 4y = 12 \\ -9x + 12y = 6 \end{cases}$

e) $\begin{cases} y = \dfrac{3}{4}x - 1 \\ .3x + 1.7y = 2 \end{cases}$

Computer Lab #5 – Absolute Value Sliders

1. Once you have opened the program Geometer's Sketchpad, click on menu **File/Open**, and choose file entitled *Absval4.gsp*. When the sketch on the screen opens up, you should see a sketch like the one on the right. There is a coordinate axis with three "*sliders*" you will use to change the value of the parameters m, h, and k. These three parameters control the V-graph in red that is displayed on the screen. Displayed in the upper left hand portion of the sketch under the general form of the V-graph $y = m|x - h| + k$ is the equation of the graph shown.
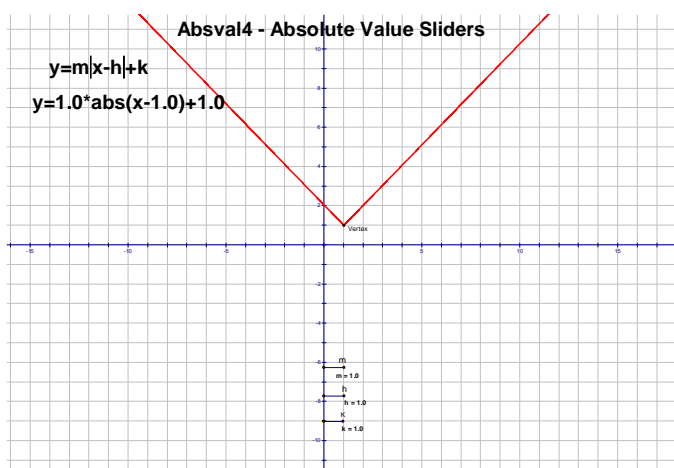
General Instructions: For the purpose of this worksheet, the accuracy for these parameters has been purposefully chosen to be one decimal place. It is important for you to realize this limitation on the accuracy, so that for certain problems you will check your answers by hand calculations in order to make any necessary small adjustments. Technology is a marvelous tool, but always know the limitations of the technology you are working with. You will be dragging on the points along the sliders that are labeled m, h, and k. This will change the values of these parameters and thus change the red V-graph in the sketch whose equation is determined by these parameters in the form $y = m|x - h| + k$.

2. Click and drag on the point labeled m.

a) Describe below what happens to the V-graph when m becomes increasingly positive.

b) What happens to the V-graph if m equals zero?

c) Describe below what happens to the V-graph when m becomes increasingly negative.

d) Summarize the role m plays in the equation $y = m|x - h| + k$.

3. Drag the slider *m* so that its value is back to being 1.0. Now drag slider point *k* and observe. Describe in the space below what happens to the V-graph as *k* changes. Be as specific as possible.

4. Drag slider point *k* back to 1.0. Now drag slider point *h* and observe. Describe in the space below what happens to the V-graph as *h* changes. Be as specific as possible.

5. Write the equation in the *Vertex Form* $y = m|x - h| + k$ for each of the V-graphs described below. As a check, drag the *m*, *h*, and *k* slider points to the appropriate values so that the graph described is drawn on the screen. Remember, the accuracy of the slider values is only to one decimal place. You should do paper and pencil calculations first, and then use the slider drawing as a check to make sure you have the correct equation of the V-graph.

a) Vertex at (−1,2) and contains the point (0,4).

b) Vertex at (2,3) and contains the point (4,0).

c) Vertex at (−3,−1), and is the same shape as $y = 3|x - 2| + 5$

d) *x*-intercepts at (4,0) and (−4,0), and contains the point (1,−2).

e) *x*-intercepts at (6,0) and (−2,0), and contains the point (5,1).

f) Has the same vertex as $y = 3|x - 2| + 5$, and contains the point (0,0).

Computer Lab #6 – Parabolas – Vertex Form

After starting the program Geometer's Sketchpad, choose Open under the File menu, and choose the file **Parabvtx4.gsp**. When the file opens up, you should see a picture of a parabola on an axis system that looks like the one at right. The three sliders are used to change the values of *a*, *h*, and *k* in the equation $y = a(x-h)^2 + k$.
The diagram will change dynamically as you change the value of the three parameters.
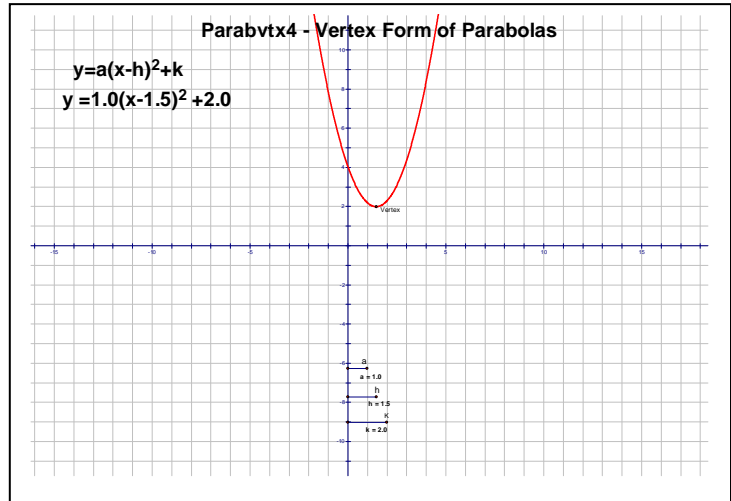
1. To change the value for any of the three parameters, simply click on the point labeled *a*, *h*, or *k*, and then slide it back and forth on the horizontal line containing that point. Try it by changing the value of *a* while leaving *h* and *k* at 0.0.

a) What happens to the curve when the value of *a* increases so that it is greater than one?

b) What happens to the curve when the value of *a* decreases, but stays positive?

c) What happens to the curve when the value of *a* becomes negative?

d) What happens to the curve when the value of *a* becomes zero?

2. Change the value of *h* by dragging on the appropriate slider.

a) What happens to the curve when the value of *h* changes? Describe in reasonable detail.

b) Change the *h* and *a* sliders so as to display the parabola with vertex (2,0) and *y*-intercept (0,8). Write down an equation for this parabola in vertex form.

c) Use the sliders to display the parabola with the same shape as the one in b) above, but with vertex at (−4,0). Write an equation of this parabola, and determine its *y*-intercept algebraically.

d) Use the *a* and *h* sliders to draw the parabola opening down with the same shape as the ones in b) and c), and whose vertex is at (2,0). Write an equation of this parabola.

3. Change the value of *k* by dragging on the appropriate slider.

a) What happens to the curve when *k* changes? Describe in reasonable detail.

b) Change the *h, k,* and *a* sliders so as to display the parabola with vertex (−1,2) and *y*-intercept (0,5). Write down an equation in vertex form for this parabola.

c) Use the sliders to display the parabola with the same shape as the one in b) above, but with vertex at (5,−2). Write down an equation of this parabola, and determine its *y*-intercept even though it may not be in view.

4. First determine the equation of the following parabolas in the space below. Then, adjust the sliders appropriately to check to see if you have the correct parabola.

Vertex (1,−1), *y*-intercept (0,4).

Vertex (−4,−3), contains point (−2,−1).

Vertex (5,2), contains point (1,−6)

Computer Lab #7 – Parabolas – Factored Form

After starting the program Geometer's Sketchpad, choose Open under the File menu, and choose the file ***Parabfct4.gsp***. When the file opens up, you should see a picture of a parabola on an axis system that looks like the one at right. The three sliders are to be used to change the values of *a*, *p*, and *q* in the equation $y = a(x - p)(x - q)$. The diagram will change dynamically as you change the value of the three parameters.

Parabfct4 - Factored Form of Parabolas

y=a(x-p)(x-q)

y =2.1(x-2.8)(x-1.4)

a = 2.1

p = 2.8

q = 1.4
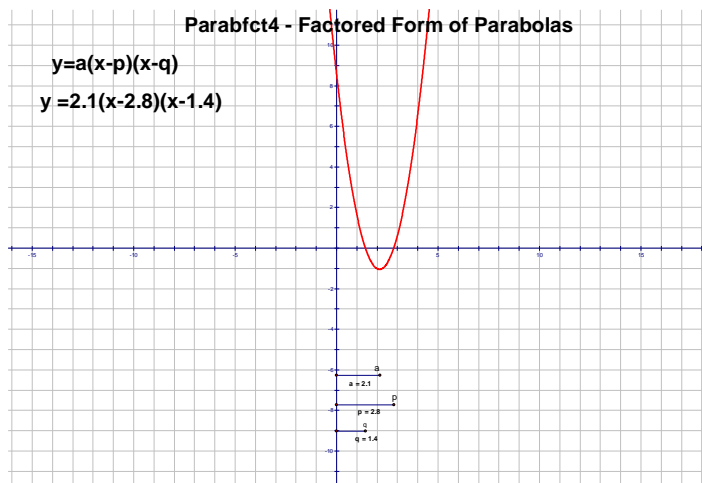
1. To change the value for any of the three parameters, simply click on the point labeled *a*, *p*, or *q*, and then slide it back and forth on the horizontal line containing that point. Try it by changing the value of *a* while leaving *p* and *q* at 0.0.

a) What happens to the curve when the value of *a* increases so that it is greater than one?

b) What happens to the curve when the value of *a* decreases, but stays positive?

c) What happens to the curve when the value of *a* becomes negative?

d) What happens to the curve when the value of *a* becomes zero?

2. Change the value of *p* by dragging on the appropriate slider.

a) What happens to the curve when the value of *p* changes? Describe in reasonable detail.

b) Change the *p* and *a* sliders so as to display the parabola with *x*-intercepts at (0,0) and (4,0), and a vertex at (2,-8) and Write down an equation in factored form for this parabola.

c) Use the sliders to display the parabola with the same shape as the one in b) above, but with *x*-intercepts at (0,0) and (-3,0). Write an equation of this parabola, and determine its vertex.

d) Use the *a* and *p* sliders to draw the parabola opening down with the same shape as the ones in b) and c), and whose *x*-intercepts are (0,0) and (6,0). Write an equation of this parabola, and determine its vertex.

3. Change the value of *q* by dragging on the appropriate slider.

a) What happens to the curve when *q* changes? Describe in reasonable detail.

b) Change the *a, p,* and *q* sliders so as to display the parabola with *x*-intercepts at (2,0) and (−2,0), with vertex at (0,4). Write down an equation in factored form for this parabola.

c) Use the sliders to display the parabola with the same shape as the one in b) above, but with *x*-intercepts at (-3,0) and (5,0). Write down an equation of this parabola, and determine its vertex even though it may not be in view. Does the computer display give a perfectly accurate value for the vertex? Explain.

4. First determine the equation of the following parabolas in the space below. Then, adjust the sliders appropriately to check to see if you have the correst parabola.

a) *x*-intercepts at (-4,0) and (6,0), and Vertex (1,-5).

b) *x*-intercepts at (-5,0) and (1,0), and containing the point (3,32)

c) *x*-intercepts at (0,0) and (-3,0), and containing the point (2,3)

Computer Lab #8 – Parabolas – Standard Form

After starting the program Geometer's Sketchpad, choose Open under the File menu, and choose the file ***Parabstd4.gsp***. When the file opens up, you should see a picture of a parabola on an axis system that looks like the one at right. The three sliders are to be used to change the values of *a*, *b*, and *c* in the equation $y = ax^2 + bx + c$. The diagram will change dynamically as you change the value of the three parameters.

Parabstd4 - Standard Form of Parabolas

y=ax²+bx+c

y =2.1x² +2.8x +1.4

a = 2.1

b = 2.8

c = 1.4
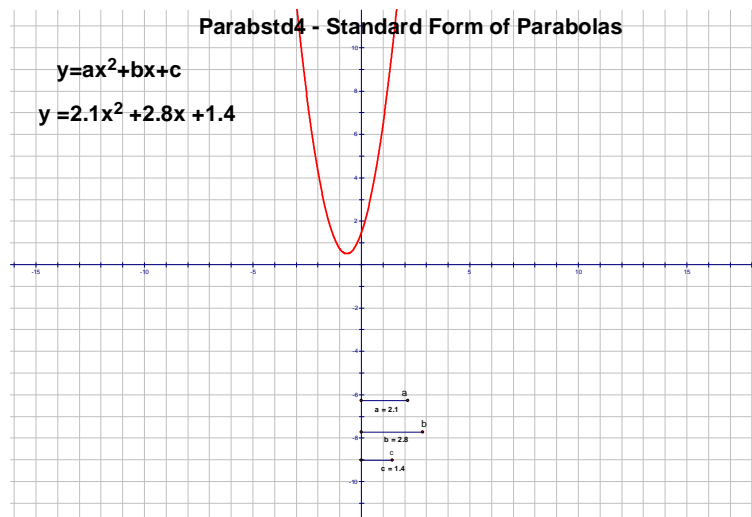
1. To change the value for any of the three parameters, simply click on the point labeled *a*, *b*, or *c*, and then slide it back and forth on the horizontal line containing that point. Try it by changing the value of *a* while leaving *b* and *c* at 0.0.

a) What happens to the curve when the value of *a* increases so that it is greater than one?

b) What happens to the curve when the value of *a* decreases, but stays positive?

c) What happens to the curve when the value of *a* becomes negative?

d) What happens to the curve when the value of *a* becomes zero?

2. Change the value of *c* by dragging on the appropriate slider.

a) What happens to the curve when the value of *c* changes? Describe in reasonable detail.

b) Change the *c* and *a* sliders so as to display the parabola with a vertex at (0,2) and an *x*-intercept of (1,0). Write down an equation in standard form for this parabola.

c) Use the sliders to display the parabola with the same shape as the one in b) above, but with vertex at (0,5). Write an equation of this parabola, in standard form.

d) Use the *a* and *c* sliders to draw the parabola opening up with the same shape as the ones in b) and c), and whose *x*-intercepts are (2,0) and (−2,0). Write an equation of this parabola, and determine its vertex even if it is not in your viewing window.

3. Change the value of *b* by dragging on the appropriate slider.

a) What happens to the curve when *b* changes?. Describe in reasonable detail.

b) Change the *a, b,* and *c* sliders so as to display a parabola with axis of symmetry at $x = 2$ and passing through the origin. There are an infinite number of examples to choose from, write down an equation in standard form for two of them.

c) Use the sliders to display the parabola with the same shape as $y = 0.5x^2$, but with vertex at (−1,4). Determine the x-intercepts for this parabola..

4. First determine the equation of the following parabolas in the space below. Then, adjust the sliders appropriately to check to see if you have the correct parabola.
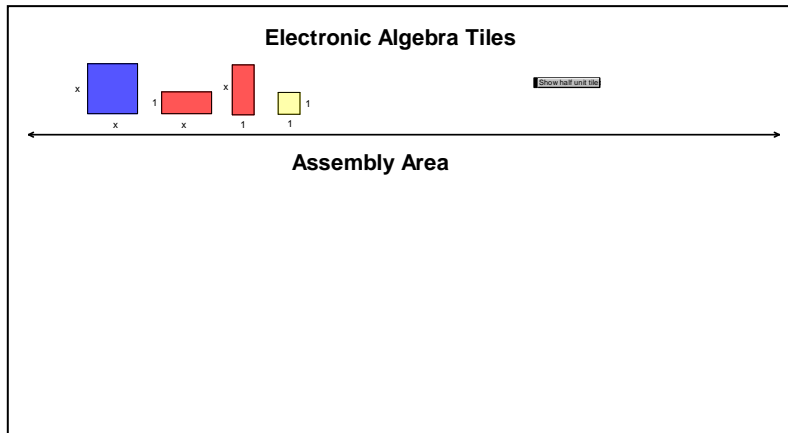
x-intercepts at (-1,0) and (3,0), and Vertex (1,2).

Vertex at (-2,3), and contains the point (0,11).

Contains the three points (0,−4), (1,-1), and (2,1).

Computer Lab #9 – Factoring with Electronic Algebra Tiles

1. This lab uses electronic algebra blocks to assist in the understanding of some of the basic ideas behind factoring. Start the GSP program and open the file entitled *factor4.gsp*. When the sketch window opens up, you should see the sketch at right. The resources for this activity are the building blocks at the top of the window. They consist of: a blue x by x square called an x^2–**block,** a red 1 by x rectangle called an **x-block**, and a yellow 1 by 1 square called **a unit block**. The x-block is given in two different orientations for reasons that should be apparent later. The basic principle at work in this exercise will be *that the area of the whole is equal to the sum of the areas of the parts*.

2. How to proceed is best explained through an example. We want to use one x^2-block and two x-blocks. To obtain the needed blocks in the assembly area, click on the middle of the x^2-block and choose Copy and then Paste from the Edit menu. A copy of the x^2-block should appear. Simply click and drag on this block to move it anywhere you want in the assembly area. Ctrl-C to copy, and Ctrl-V to paste is the keyboard shortcut to this process. Make two copies of the x-block, so that you now have the x^2-block and two x-blocks in the assembly area. Notice the dimensions do not move with the blocks. If you want dimensions in the assembly area, the x and 1 must also be copied and pasted. Your job now is to arrange these blocks so that they form a rectangle. **This goal of forming a rectangle from the given building blocks is the key to this entire worksheet**. There is more than one way this could have been done, (a few different examples are shown at right), but no matter what rectangle you formed, its dimensions should be x by x+2. If we now write an expression for the area of the rectangle and set it equal to the sum of the areas of the individual building blocks that formed the rectangle, we get $x^2 + 2x = x(x+2)$. This right side of this equivalency represents the factored form of the polynomial $x^2 + 2x$.

3. Summary of procedures.
a) Make enough copies of basic building blocks for the given problem.
b) Assemble blocks so that they form a rectangle. Make sure you select the borders as well as the interior of the block before copying and pasting.
c) Write an equivalency in which the left side is the sum of the areas of parts, and the right side is the
     area of the rectangle expressed in the form (*length*)(*width*).
d) Any extra building blocks can temporarily be dragged out of the way for possible use later.

4.   Let's do one more example that is a little more involved. This time our given building blocks are one x^2-block, four x-blocks, and three unit blocks. This time, the rectangle that
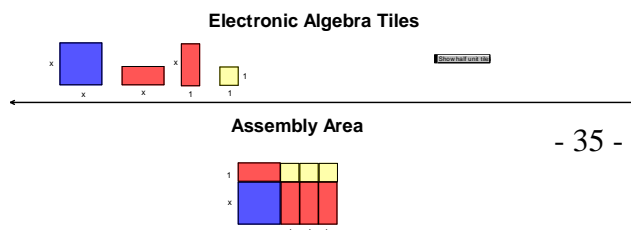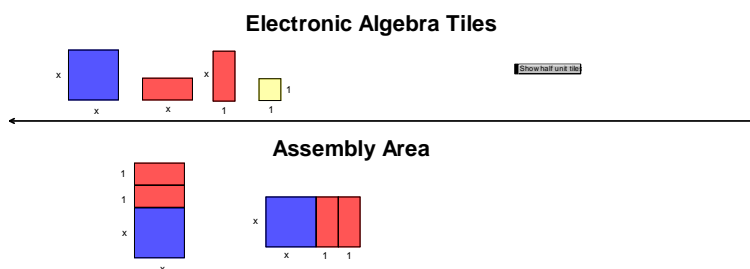
is built has dimensions x+1 by x+3. The equivalency that gives the factoring is $x^2 + 4x + 3 = (x+1)(x+3)$. The picture is shown at right.

5.   Do the following examples. Use the space provided to copy the sketch of the rectangle, and to write out the equivalency that gives the factored form of the polynomial.

a) two x^2-blocks, one x-block

b) one x^2-block, five x-blocks, six unit blocks

c) two x^2-blocks, five x-blocks, two unit blocks

d) two x^2-blocks, five x-blocks, three unit blocks

e) three x^2-blocks, eight x-blocks, four unit blocks

6.   Make up your own example. It must involve one x^2-block, six unit blocks, and your choice for the number of x-blocks. How many different examples are there? Write all the appropriate equivalencies in the space below.

Computer Lab #10 – Completing the Square with Electronic Algebra Tiles

This lab uses virtual algebra blocks to assist in the understanding of some of the basic ideas behind factoring. Start the GSP program and open the file entitled *factor4.gsp*. When the sketch window opens up, you should see the same sketch as in Computer Lab #9 on factoring. The resources for this activity are the building blocks at the top of the window. They consist of: a blue x by x square called an **x^2–block,** a red 1 by x rectangle called an **x-block**, and a yellow 1 by 1 square called **a unit block**. The x-block is given in two different orientations for reasons that should be apparent later. The basic goal of these exercises will be to determine the correct number of unit blocks that are needed in order to make a *square* from the given building blocks.

2. How to proceed is best explained through an example. If we are given one x^2-block and four x-blocks, we need to determine the number of unit blocks necessary in order to form a square. Although there is more than one way your blocks could be arranged, all of them require four additional unit block, and they all result in a square with exactly the same dimensions. Two different arrangements will appear in your sketch by clicking on the button **Show completing the square example** with the Selection Tool. If we now write an expression for the area of the square and set it equal to the sum of the areas of the individual building blocks that formed the square, we get $x^2 + 4x + 4 = (x+2)^2$. Click the button **Hide completing the square example** to hide this example again.

3. Do the following examples. Use the space provided to copy the sketch of the square, and to write out the equivalency that gives the factored form of the polynomial into a perfect square.

a) one x^2-blocks, six x-blocks

b) one x^2-block, eight x-blocks

c) four x^2-blocks, four x-blocks

4. What happens if the number of x-blocks is not an even number? This causes problems that can be illustrated if you try to complete the square with one x^2-block and three x-blocks. The way around this problem is to have the x-block and unit block broken into smaller pieces. By having the x-block available in two equal rectangular pieces with dimensions 0.5 by x, and by having the unit block broken into four smaller squares with dimensions 0.5 by 0.5, this problem can be solved. The illustration of completing the square for one x^2-block and three x-blocks can be seen by clicking on the button **Show example of completing the square with half blocks** with the <u>Selection Tool</u>. The square has been assembled using one x^2-block, two x-blocks, two half x-blocks, one unit block, and five quarter blocks. (A quarter block is the 0.5 by 0.5 block that has a quarter the area of a unit block). The equivalency is thus $x^2 + 3x + \dfrac{9}{4} = \left(x + \dfrac{3}{2}\right)^2$. Click the button **Hide example of completing the square with half blocks** in order to hide this example again.

5. In order to obtain the additional building blocks needed for exercises when there is an odd number of x-blocks, click on the button **Show half block tiles** with the Selection Tool.Using these additional building blocks, complete the square for the following examples. Use the space provided to copy the sketch of the square, and to write out the equivalency that gives the factored form of the polynomial into a perfect square.

a) one x^2-block, one x-blocks

b) one x^2-block, five x-blocks

6. Suppose you take one x^2-block and P x-blocks. Predict how many unit blocks are needed to complete the square. Complete the following: $x^2 + Px + \left(\right)^2 = \left(x + \underline{}\right)^2$

Computer Lab #11 – Vertical and Adjacent Angles

1. Open a new sketch, and using the <u>Segment Tool</u>, construct two intersecting line segments *AB* and *CD*, and their point of intersection E as shown in the sketch below. (Remember there are three ways to get the point of intersection. Click on the intersection with the <u>Selection Tool</u>, click on the intersection with the <u>Point Tool</u>, or select both line segments and choose **Point of Intersection** from the **Construct** menu)
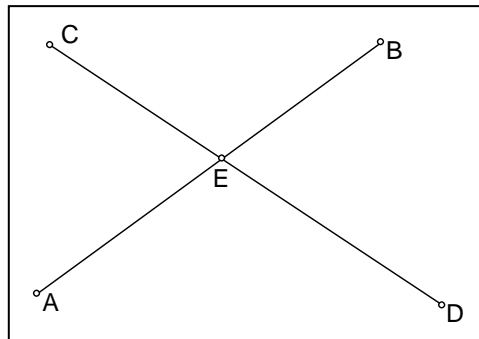
2. Measure angles CEA, AED, and DEB. (Select 3 points in order to measure an angle.) Open the Calculator from the **Measure** menu, and display the sum of angles CEA and AED by clicking on those measurements with a plus sign in between. Now, drag on the end points of the line segments and on the segments themselves and observe what happens to the measures of the angles and their computed sum.

3. From your observations above, write how you would predict the measure of angle CEB from knowing just the measure of angle AEC._____. Write how you would predict the measure of angle CEB from knowing just the measure of angle AED. _____.

4. Angles CEA and AED are adjacent angles in that they share a commion vertex, and there is something else that is important about them. Name another pair of angles that share the same property as these two. _____. Pairs of angles like CEA and AED are called a *linear pair*. Try and write a definition of *a linear pair* in the space below.

5. Angles CEA and BED are defined to be ***vertical angles,*** (also called ***opposite angles*** sometimes). Name another pair of vertical angles. _____. Try and write a definition of vertical angles in the space below.

6. Write a sentence or two summarizing what you have observed in this lab about vertical and adjacent angles of intersecting line segments. Although we have used line segments to illustrate these relationships, the same angle relationships are true for any two intersecting straight objects including segments, rays, and lines.

Computer Lab #12 - Parallel Lines crossed by a Transversal

1.   Use the <u>Segment Tool</u> to draw a line segment AB across the width of the sketch window. Now draw a segment CD diagonally across the window as shown below. Construct their intersection point E. Use the <u>Point Tool </u>to put a floating point F on CD, and then construct the line through F parallel to AB. Put two floating points G and H on this line as shown below.
.
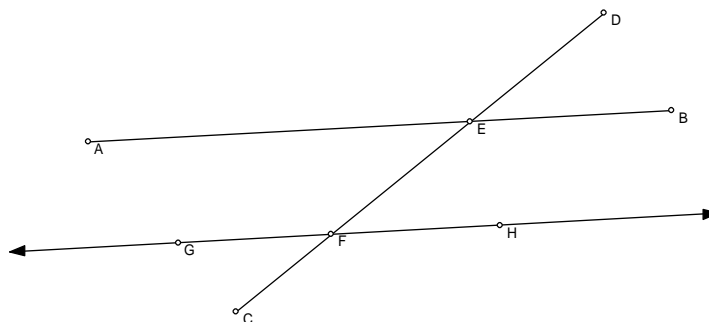
2. Think of the diagram as two parallel roads AB and GH being crossed, (or transversed), by another road CD, thus forming two intersections at E and F. CD is thus referred to as a ***transversal***. Measure the four angles at the E intersection. Vertical angles being equal, and adjacent angles being supplementary should be no surprise given our first lab on vertical and adjacent angles. Measure the four angles at the F intersection also. Arrange the angle measures at the two different intersections so that they "**correspond**". Now, drag on the objects in your sketch and observe these measures.

3. Angles DEB and EFH are defined to be ***corresponding angles.*** There are three other pairs of corresponding angles in the figure. Try and name them. _____
Write a definition of corresponding angles in the space below, and what you have observed about them.

4. Angles AEF and HFE are defined to be ***alternate interior angles***. The word alternate comes from the fact that they are on alternate sides of the transversal CD. Interior refers to the fact they are in the interior region of the parallel lines. There is another pair of alternate interior angles. Can you name them? _____. Write a definition of alternate interior angles in the space below, and what you have observed about them.

5. Angles BEF and HFE are defined to be ***interior angles on the same side.*** Name another pair. _____. Write a definition of interior angles on the same side in the space below, and what you have observed about them.


6. What pairs of angles do you think are referred to as ***alternate exterior angles***? What property do they have?

Computer Lab #13 - Sentry Theorem

You have just been hired as a **sentry** of an important quadrilateral building. The job is to walk around the building close to every wall and turning at each corner. Through how many degrees in total does the sentry turn during one complete trip.

1. Hold down the shift key, and use the point tool to plunk down 4 random points in clockwise order so that they will all be selected when you are finished.

2. Now, change the straightedge tool to **Ray,** and while all points are still selected, choose **Ray** from the **construct** menu.

3. You now have a quadrilateral with all 4 sides extended. Using the point tool, construct a point on each of the 4 rays, outside the quadrilateral. Construct, or relabel the points, so that your diagram looks like the sketch to the right. Also, make sure your quadrilateral building is *convex*, in other words, none of the interior angles is greater than 180 degrees.
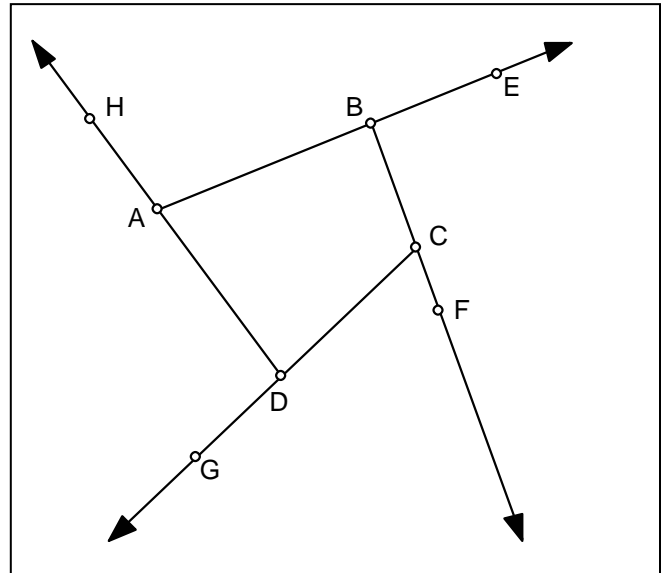
4. Let's assume that the sentry is going to start at vertex D, and walk clockwise to vertex A, turn through angle HAB and then proceed towards vertex B. What three angles do we want to measure **in addition to angle HAB** in order to find the total # of degrees turned during our trip back to our starting position and facing the same initial direction? ∠_____ , ∠ _____, and ∠_____.

5. If you answered EBC, FCD, and GDA, you are right. Measure the 4 angles, select all 4 and go to **Calculate** in the **Measure** menu, and find their sum.

6. Now click on any of the vertices of the quadrilateral and drag to change its shape, but keep the quadrilateral *convex*. What do you notice? _____

7. Now, let the quadrilateral become concave. Does anything change?_____.
Do you have an explanation of your observation?_____.

8. Repeat all the steps above for a 5 sided figure. Did you anticipate the result that occurred ?_____. Try and make a general, but clear statement below. We will call this the *Sentry Theorem*.

Computer Lab #14 - The Angle Bisector Theorem

1. Construct a random triangle ABC and bisect angle A. (Select the three points defining the angle and choose **Construct/Angle Bisector**). Construct the point of intersection on side BC and label it D. Measure the distances from A to B and A to C. Also measure the distances BD and DC. Remember, to measure a distance between two points, select the two points before choosing the **Measure** menu. Use the calculator to display the ratio BD/DC.

$$\frac{BD}{DC} = 0.495$$

B BD = 3.407 cm

D

AB = 5.664 cm

DC = 6.888 cm

A

AC = 11.449 cm

C
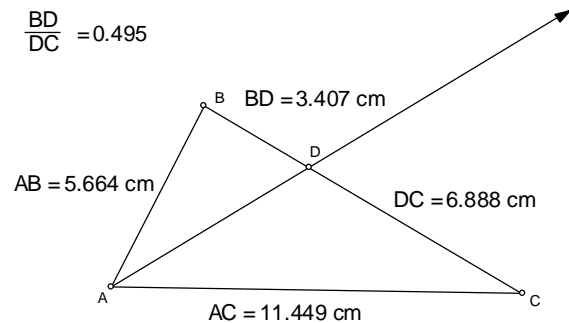
2. Notice that D does not necessarily intersect BC at its midpoint. Drag on the vertices to change the shape of the triangle and notice point D moves as well.

What do you notice about the lengths of sides AB and AC when D is closer to B than it is to C?

What do you notice about the lengths of sides AB and AC when D is closer to C than it is to B?

What do you notice about the sides AB and AC when D is close to the midpoint of BC?

3. Now, drag on the vertices so that AB is approximately twice as long as AC. What do you notice about the position of D and in particular the ratio BD/DC? _____

What if AB is three times as long as AC? _____

What if AB is half as long as AC? _____

4. Can you make a conjecture about the specific location of point D in relation to the relative sizes of sides AB and AC?

5. Let's take a more formal look at what is going on when you bisect an angle in a triangle, and see if your conjecture proves correct.

a) Using your existing triangle ABC. Construct lines through B and C that are parallel to AD, and let E be the point where one of these parallel lines intersects the extension of CA. (You need to construct ray CA). It might be helpful to thicken and change the color of the three parallel lines. Hide the previous mesurements. Your sketch should look like the one at right.
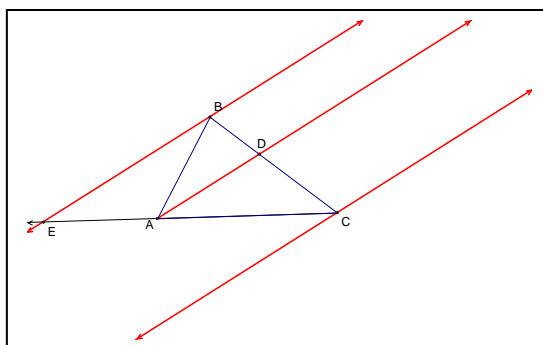
b) Identify three other angles within the interior of triangle BEC that are equal to angle BEC. Remember that AD is an angle bisector. State your reasons why the four angles are equal.

c) State as a proportion how the 4 lengths CA, AE, CD, DB are related to each other. Give a brief explanation as to why.

d) The only length used in your last statement that was not part of the original triangle is length AE. What length in the original triangle will always be equal to AE and why?

e) Hopefully your answer to the question above was AB = AE because ABE is an isosceles triangle. Substitute AB for AE from step 3 and write a statement relating the lengths AB, AC, BD, and DC. *This is the angle bisector theorem.* How does it compare with your conjecture on the previous page?

6. Try and state in words alone, without using any symbols, what the angle bisector theorem says.

7. Use the angle bisector theorem to find the length IM in the diagram to the right. I is the incenter of the triangle. This problem can be modeled in GSP, or solved using a different method as a way to check your work.
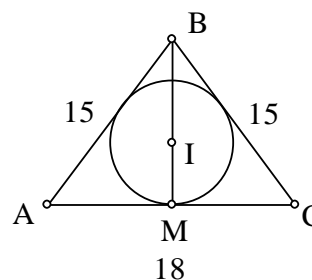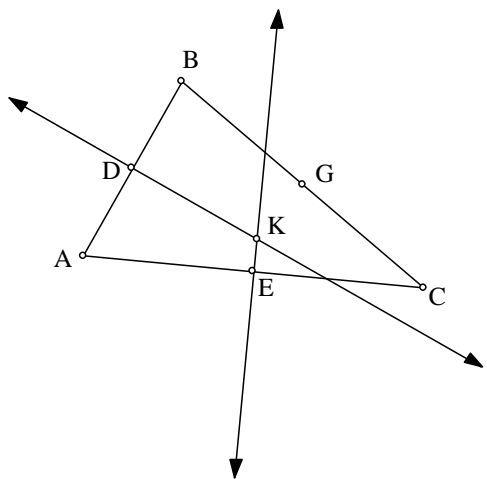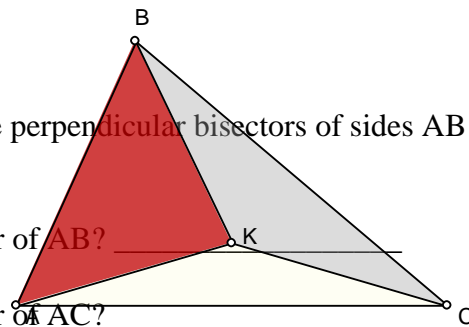
Computer Lab #15 - Circumcenter Lab

1. Start a new sketch and construct a random triangle ABC and the perpendicular bisectors of sides AB and AC. Construct their intersection point and label it K.

What must be true about K since it is on the perpendicular bisector of AB? _____

What must be true about K since it is on the perpendicular bisector of AC? _____

If KA = KB = KC, then what other line should be ***concurrent*** with the perpendicular bisectors of AB and AC that you have already constructed? _____.

To show that K is also on the perpendicular bisector of BC, construct the midpoint G of side BC and measure the angle KGB. What angle does KGB need to be, in order to confirm concurrency? _____ .
Drag the triangle around and notice the angle remains constant. Now construct the third perpendicular bisector to demonstrate that it is indeed concurrent. This point of ***concurrency*** is called the *circumcenter* because it is the center of the circle that ***circumscribes*** the triangle. Construct the ***circumcircle*** and drag the triangle around and observe what happens.

2. Hide all construction lines and measures so that only the triangle, its midpoints, and the circumcenter K remain in view. Measure the three angles of the triangle. Drag the vertices of the triangle around and note the location of the circumcenter K. Answer the following questions :

a) When is K inside the triangle? _____

b) When is K on side AB? _____ Where on side AB does K move to. _____
    When is K on side AC? _____ Where on side AC does K move to. _____
    When is K on side BC? _____ Where on side BC does K move to. _____

c) When is K outside the triangle? _____

d) Select K and choose **Trace Point** from the **Display** menu. Drag vertex A around slowly and observe the trace of point K. Describe the path of point K. _____.
3. Turn the Trace of K off by clicking on **Trace Point** from the **Display** menu. Draw segments KA, KB, and KC to form three isosceles triangles ABK, ACK, BCK. Form the polygon interiors of these triangles

and make them different colors.

a) Measure angles AKC, BKC, AKB, and compare them to angles ABC, BAC, and ACB respectively. (Put the angle measures you are comparing adjacent to each other in your sketch window to make comparing easier.) Drag the triangle around and notice their relationship. Summarize what you have observed below, and make a conjecture based upon these observations.

b) Construct rays that extend AK, BK, and CK. Construct the points of intersection of these rays with the sides of the triangle. Make whatever measurements you feel are needed to answer the following set of questions as you drag the triangle into different positions.

i) Are the shaded triangles **always** congruent? _____.
Are they **ever** congruent? _____
When ?_____

ii) Are AK, BK, and CK **always** ∠ bisectors? _____.
Are they **ever** ∠ bisectors? _____
When?_____

iii) Do the extensions of AK, BK, and CK **always** meet the opposite side at its midpoint? _____
Do the extensions of AK, BK, and CK **ever** meet the opposite side at its midpoint? _____
When? _____

iv) Are the extensions of AK, BK, and CK **always** perpendicular to the opposite side? _____.
Are the extensions of AK, BK, and CK **ever** perpendicular to the opposite side? _____.
When? _____
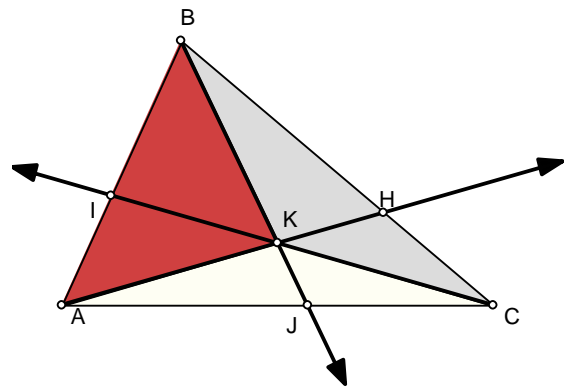
Computer Lab #16 - Median concurrency and the 2:1 centroid ratio - Part I

1. Construct a triangle ABC and let AM and BN be two of its medians. (remember to double click with the text tool on a label in order to make a change).
Label G as the point where the medians intersect.
Extend AM to the point P such that GM = MP.
(vector translation is an easy way to do this.)
Extend BN to the point Q that makes GN = NQ.
Your sketch should look like the one at right.
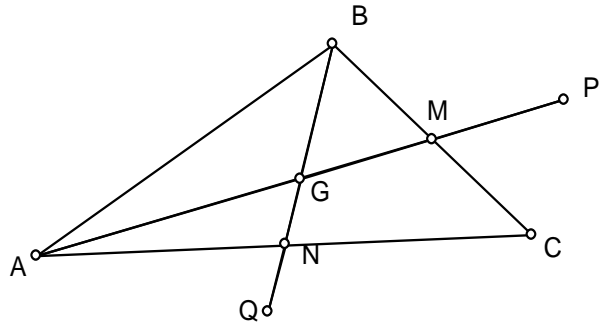
2. Connect AQCG. Change the line style of this quadrilateral to thick and color to red using the **Display** menu. Click and drag on the triangle all you want and AQCG remains what type of quadrilateral? _____. In the space below, give an explanation as to why this must be the case.

3. Connect BPCG. Change style and color to thick and blue. What type of quadrilateral is BPCG and why?

4. Explain why BG is parallel to PC, and AG is parallel to QC.

5. What kind of quadrilateral is PCQG? _____. How do you know?

6. Find two segments in the diagram that have the same length as BG. _____ , _____

7. How do lengths of segments BG and GN compare? How about AG and GM?

8. What do you think would happen if the third median CD was constructed? Try it.

9. Write in clear, concise, but generalized language what has been proven in this exercise.

Median concurrency and the 2:1 centroid ratio - Part II

1. Start a new sketch, and construct a random triangle ABC. Select the three sides of the triangle, choose **Midpoint** from the **Construct** menu. The three midpoints D,E,F are constructed and are selected. Deselect them by clicking in open space. Select vertex A and its opposite midpoint.. Click **Construct/Segment** to produce the median. Construct the median from B in a different manner. Click on the Segment Tool from the tool box. To connect point B to its opposite midpoint, place the cursor on point B, then click and drag to the opposite midpoint, then release. Your construction should now look like the one at right.
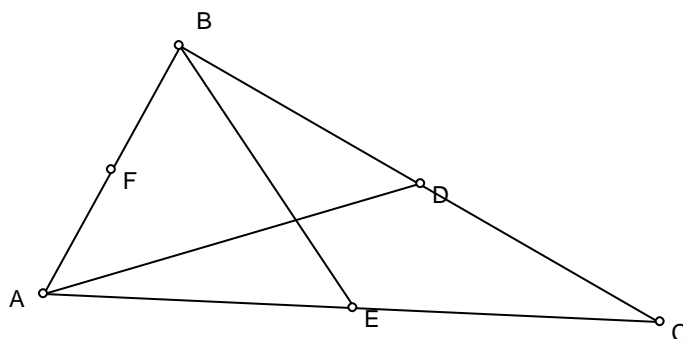
2. Although the two medians clearly intersect, the point of intersection must be constructed in order for GSP to know it exists. The easiest way to accomplish this is to simply point the Select Tool at the intersection and click. Try it. Choose **Undo** from the **Edit** menu so that you can practice the other way to construct intersections. Select the two line segments that intersect, and then choose **Point at Intersection** from the **Construct** menu. This also constructs the intersection point.

3. To verify the third median is concurrent with the other two, we want to measure the angle formed by the 3rd vertex, the intersection point of the other two medians, and the opposite midpoint. To measure the angle, you must select 3 points in the correct order, and then choose **Angle** 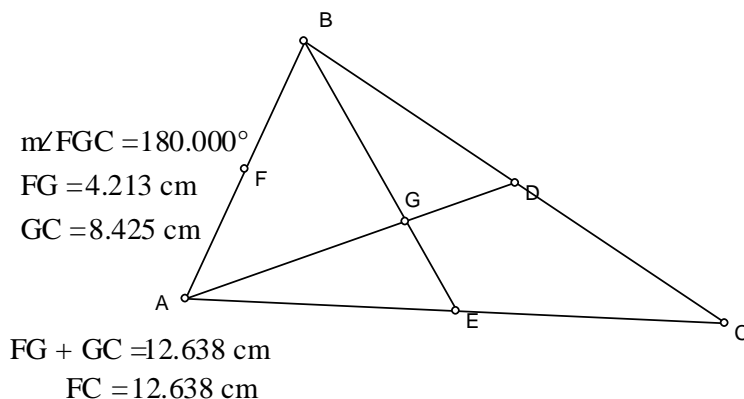from the **Measure** menu. Note that the median itself does not have to be constructed in order to measure the angle. As shown at right, the angle is exactly 180 degrees which certainly indicates the medians are concurrent.

$m\angle FGC = 180.000°$
$FG = 4.213$ cm
$GC = 8.425$ cm

$FG + GC = 12.638$ cm
$FC = 12.638$ cm

4. Another method of verifying C, G, and F are collinear is using distances. CG + GF must equal CF if the three points are collinear. To measure a distance between points, simply select the two points and choose **Distance** from the **Measure** menu. In order to have GSP add the distances, select the distance measurements CG and GF themselves and choose **Calculate** from the **Measure** menu. A calculator opens up with the selected measurements available under **Values**. Select one distance, then click on the + sign, then select the other distance. Click **OK** or hit the *Enter* key on the keyboard. The sum appears in your sketch window.

Computer Lab #17 - Fermat Point

Problem - Given the triangle A(0,0), B(5,0), C(0,7), find the point D such that the sum of the distances PA + PD + PC is a minimum.

1. Start a new sketch. Click **Edit/Preferences** and make sure will all new points are labeled automatically, that the accuracy for angles is tenths, the accuracy for distance is hundred thousandths, and the units are in cm. Choose **Graph/Show Grid.** Hide the unit point at (1,0) and use the Text Tool to click on the origin point and display the label A. Choose **Graph/Plot Points** and plot B and C given above. Click **File/Document Options**. A dialogue box opens up that enables you to break this sketch up into separate pages that are easy to access. Replace the 1 in the top box under Page Name with the name *Fermat1*. Now, in this same dialogue box, click **Add Page/Duplicate** and choose *Fermat1* when the drop down menu appears. Change the name under Page Name to *Fermat2* and click OK. This creates a duplicate page to the one you just created named *Fermat2*. Look in the lower left hand corner of your sketch, and you will see two tabs with the names *Fermat1* and *Fermat2*. Multiple pages in a sketch are created this way, and the tabs are used to move back and forth among these pages.

In the sketch *Fermat1* put a floating point D in the interior, and connect D to A, B, and C. Calculate the sum of the distances to the three vertices, and also display the measure of angles ADB, ADC, BDC. Now drag point D around until it is located at a position that minimizes the sum of the distances to the vertices. You can use the arrow keys for fine tuning the search. What do you observe about the measure of the three angles displayed as you get closer to the minimum distance? _____.
Hopefully you have noticed that the three angles are reasonably close to being equal with each 120 degrees. Note that this point D, called the ***Fermat point,*** (or *Steiner Point*), is not any of the special points of the triangle that you have probably worked with previously. (i.e. it is **not** the Centroid, Circumcenter, Orthocenter, or Incenter). Write down the coordinates of point D, and the sum of the distances to the vertices.
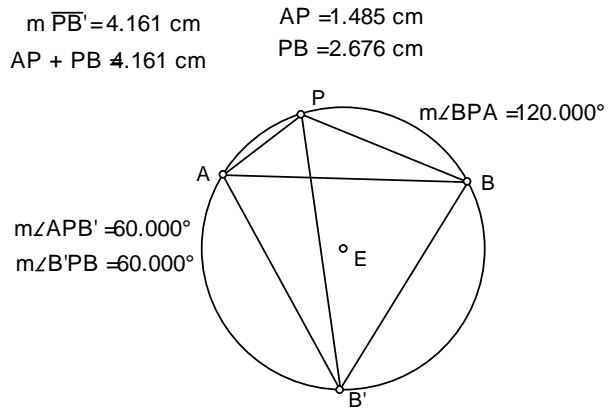
2. There is a better way to obtain the Fermat point without dragging a point around and eyeballing the distances and angles. First, let's revisit an earlier problem. Start a new sketch. Since **120 degree** angles are clearly important, we need to know how to construct the arc of the circle that contains the points that form a 120 degree angle with a line segment AB. Open a new sketch and construct a random line segment AB. Mark one end point as a center of a transformation by double-clicking on it, and then use the **Transform** menu to rotate the other end point 60 degrees. Connecting the vertices now gives you an equilateral triangle. Using perpendicular bisectors, find the ***circumcenter*** of this triangle, and construct the ***circumcircle***. Hide the perpendicular bisectors and midpoints used to construct the circumcenter. Your sketch should now look like the one to the right. This construction is the key to finding the Fermat point of a triangle. Choose **Select All** from the **Edit** menu, and then Create New Tool from the Custom Tool drop down menu. (See page 10 of this document). Name this new custom tool *circumcircle* and notice that if you view the script for this custom tool, this tool requires only two points in order to construct a triangle and its circumcircle.
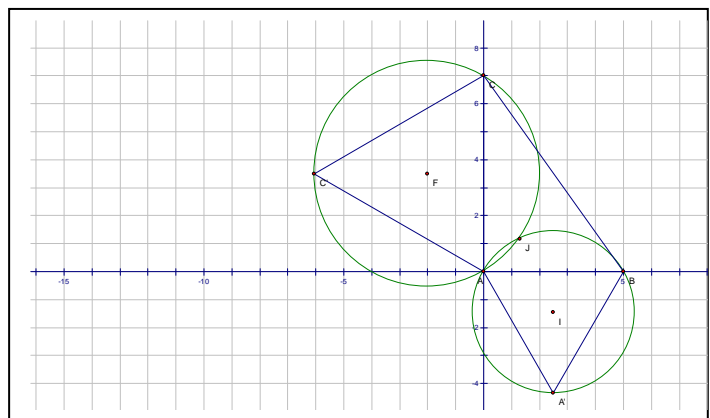
Put a floating point P on the minor arc AB. Connect P to A and B, and measure the angle APB. Can you explain why any position of P on this arc yields a 120 degree angle?

Connect P to B' and measure the angles APB' and BPB'. Notice that they are both always 60 degrees. Why?

Now measure the sum of the lengths AP + BP and compare to length PB'. Drag P around. Interesting result isn't it? *Ptolemy's Theorem* says AP*BB' + BP*AB' = AB*PB'. This is one way to prove AP + BP = PB'. Go to http://www.cut-the-knot.org/proofs/ptolemy.shtml to see the proof. Your sketch should now look like the one at the right. Keep this sketch open so the tool *circumcircle* tool is available.

m $\overline{PB'}$ = 4.161 cm
AP + PB = 4.161 cm

AP = 1.485 cm
PB = 2.676 cm

m∠BPA = 120.000°

m∠APB' = 60.000°
m∠B'PB = 60.000°

3. Click on Window and go back to the previous sketch that we created with the two tabs called *Fermat1* and *Fermat2*. Click on the tab for *Fermat2*. Now you are going to **construct** the Fermat point using the custom tool *circumcircle* that is now available to you from the Custom Tool box. Using the custom tool *circumcircle*, click and drag between points A and B. Be careful of the order you choose the points, because you want the equilateral triangle to attach to the outside of triangle ABC, not the inside. If you chose wrong, use **Undo** from the **Edit** menu, and do over. Repeat this with vertices A and C. Your sketch should look like the one above. Where does the Fermat point have to be? Hopefully you answered at the intersection of the two circles. Construct this intersection point D, and connect this point D to A,B, and C in order to measure the angles and sum of the distances. Compare answer to answer from *Fermat1* in which we simply eyeballed point D.

Computer Lab #18 - RGB Lab with Ceva's Theorem

1. Construct a random triangle, and relabel the vertices R,G, and B. Put a floating point P on the inside of the triangle, and connect P to the three vertices to form the three triangles PGB, PRB, and PRG. Construct the three polygon interiors for those triangles, measure their areas, and color them red, green, and blue respectively.(i.e. PGB = red, PRB = green, and PRG = blue.) It is important the colors match the correct triangle. Note that each triangle has the color beginning with the letter of the vertex not contained by that triangle. You might want to change the name of the triangle interiors to match their color, (as I have done in the sample sketch at right). You do this by using the Text Tool and double-click on the area measure for each triangle. As you drag P around, note that the closer point P is to a particular vertex, the more of that color shows up in the triangle. Therefore, every position of P can be thought of as a mixture of a certain proportion of red, green, and blue color. Drag P around and experiment.
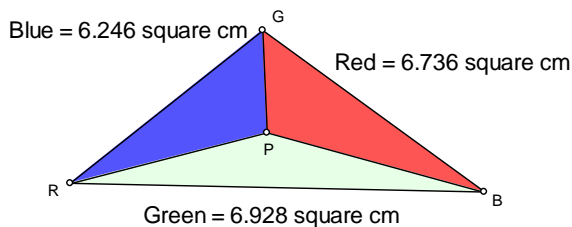
2. To display the fractional amount of red, green, and blue, compute the total area of the triangle and then divide the colored regions by the total to display their respective percentages as decimals. What would be the proportions for red, green, and blue in that order if P were located at the following points:
a) the midpoint of RB _____   b) the midpoint of RG _____
c) at vertex B _____   d) the centroid _____.

3. Change to the *line tool*, and construct the three lines PR, PG, and PB. Construct the points of intersection of each line with the side of the triangle. Label those points X,Y, and Z where X is on RG, Y is on GB, and Z is on BR. Hide the lines, and construct the line segments PX, PY, and PZ. Measure the distances RX, GX, GY, BY, BZ, and RZ. The line segments RY, GZ, and BX are referred to as *cevians* of the triangle. Your sketch should look like the one to the right.
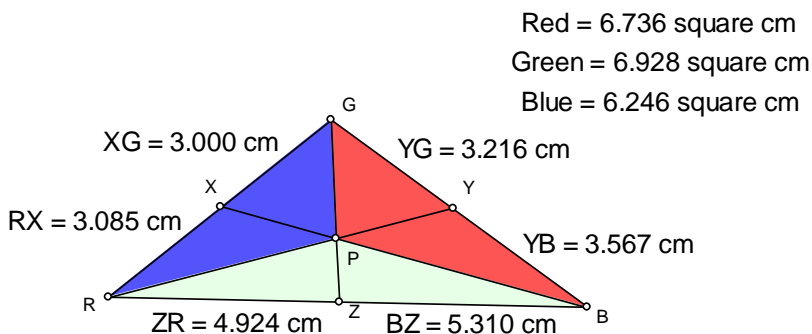
4. The ratio of sides RZ to BZ is the same as the ratio of areas of what colored triangles?

   Write a clear and concise statement in the space below explaining why this is true.

5. Now complete the chart below by filling in the color of the triangles in the second row.

Ratio of Sides	RZ to ZB	BY to YG	GX to XR
Ratio of Triangle Area			

6. Use the computer calculator to compute the following product of ratios. $\dfrac{RZ}{ZB} * \dfrac{BY}{YG} * \dfrac{GX}{XR} = ?$

Drag the triangle around to see if this product changes. What must be true about P for this product to remain constant?

Substitute the color of the triangle area from the chart into the product of the ratios listed above. Is it clear why this product is always the constant that was computed?

This result is known as **Ceva's Theorem.** Try and write down in the space below a clear sentence that sums up what this theorem says any three concurrent *cevians* in a triangle.

Computer Lab #19- The Orthocenter

1. Start your sketch with a random triangle ABC. It is given that triangle ABC is the midpoint triangle of triangle DEF. (i.e. A,B, and C are the midpoints of the sides of triangle DEF).

2. Construct triangle DEF and color the lines red. Keep in mind the properties that the midpoint triangle must have. (Hint: the Midline Theorem is certainly going to come into play here)

3. Construct the perpendicular bisectors of triangle DEF and color then blue. Construct the point of intersection. Your diagram should now look like the one at right.
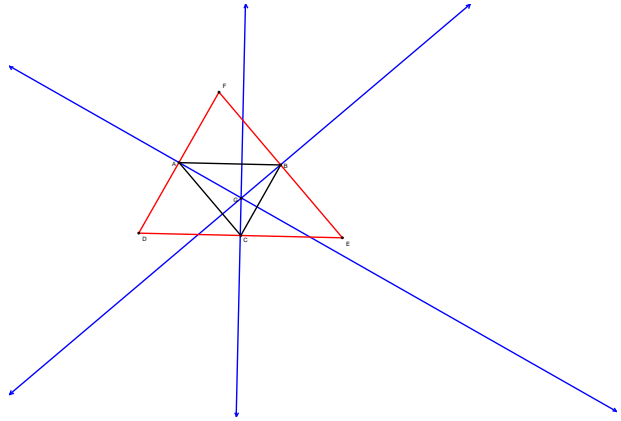
At what special point **of triangle DEF** are these perpendicular bisectors *concurrent?*
_____.

4. The same lines that are the perpendicular bisectors of triangle DEF are also special lines for the original triangle ABC. What is the name for these special lines **in triangle ABC**. _____.

5. Complete the following. The *perpendicular bisectors* of triangle DEF are concurrent at the
_____of <u>that</u> triangle. These same three lines are also the _____ of triangle ABC, and that same point of concurrency is called the *orthocenter* of triangle ABC. Relabel this point *orthocenter.*

6. Hide everything from your sketch except the original triangle ABC, and the *orthocenter*. Drag on the vertices and notice what happens to the *orthocenter* for various types of triangles.

When is the orthocenter inside the triangle? _____

When is the orthocenter on the triangle? _____. Where is it located? _____

When is the orthocenter outside the triangle? _____

Reviewing Some Important Points of Concurrency in a Triangle

a) **Circumcenter** - intersection of the three **perpendicular bisectors**. Center of circumscribed circle.
b) **Centroid** - intersection of the three **medians**. Balancing point or center of gravity for triangle.
c) **Orthocenter** - intersection of the three **altitudes**.
d) **Incenter** - intersection of the three **angle bisectors**. Center of inscribed circle.
e) **Fermat point** - intersection of three lines extending from vertex to opposite vertex of attached equilateral triangles. 120° angles formed by segments connecting to vertices.
Minimizes sume of the distances to the three vertices.

As a nice review exercise, construct all five of these special points on the random triangle ABC that you just used to locate the orthocenter on the previous exercise. Hide all construction lines after each one so to keep the sketch uncluttered. Label points clearly with their correct name, and drag on vertices to watch them move around. Which of these points were constructed with *cevian lines*, and thus the six segments formed by those construction lines intersecting the sides of the triangle will satisfy Ceva's Theorem? (See Lab #18 pages 43-44)
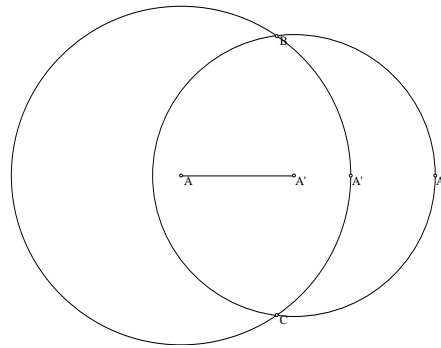
Your sketch should look like the one below.
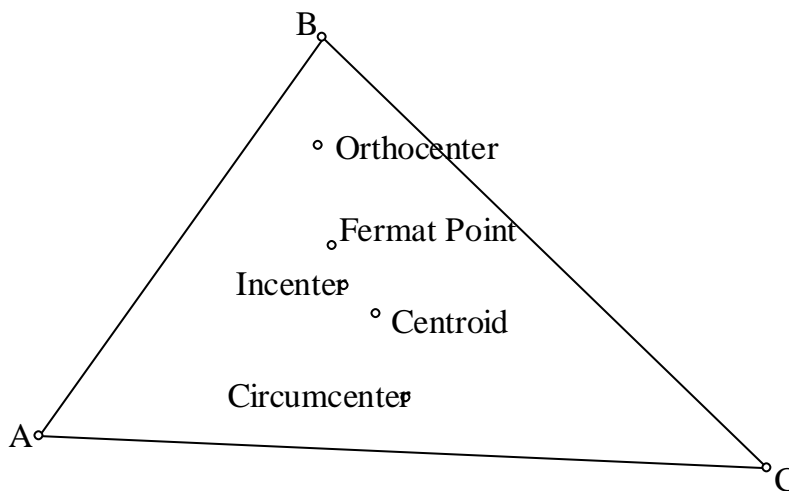
Three of these five points are always *collinear*. Can you name the three. The line they are on is called the **Euler Line**. There is also a special distance relationship maintained between these three points for all triangles. Once you have determined which three points are collinear, measure the distances between them and try and discover this relationship. Summarize your conjectures below.

Computer Lab #20 - Exploring Congruence Using Constructions

I.   SSS Given: a =5 cm, b = 6 cm, c = 4 cm
Make sure you are working in cm by checking **Units** under
the **Edit/Preferences** menu.
Start a new sketch, and construct a single point A. Select A,
and translate A, 4 cm at an angle of   0 degrees. Connect A
and A' and you have one side of the triangle and two of the
vertices. The third vertex must be 5 cm from A', and 6 cm
from A. We need circles centered at A' and A with the
appropriate radii. Select A' and translate 5 cm in any direction
to form point A". Now, construct the circle centered at A' going through A". Repeat the process to get a
circle centered at A with radius 6 cm. The two circles intersect at two points. Construct the points of
intersection of the two circles. Your sketch should look like the one to the right. Although there are two
triangles possible, they are mirror images of each other about the line segment AA', so they are clearly
congruent. Construct just one of the triangles, and hide the circles and the extra points. Now drag on the
vertices and edges of this triangle to convince yourself that it is a rigid construction. Measure the three
interior angles, and compare them to others in the class. Has everyone constructed exactly the same
triangle. Are all 4 cm, 5 cm, 6 cm triangles exactly alike? Can you see what would happen if you try and
constructed a 4 cm, 5 cm, 10 cm triangle? Explain.

II.   SAS Given: c = 4 cm, $\angle$ A = 50°, b = 7 cm. ($\angle$ A is between the two given sides)
Start a new sketch, and once again translate an initial point A , 4 cm at an angle of 0 degrees, and relabel
the point B. This time, translate A 7 cm at an angle of 50 degrees, and relabel the point C. Connect A,B,
and C, and there is your triangle. Now drag on the vertices and edges of this triangle to convince yourself
that it is a rigid construction. Measure the other two interior angles and the third side, and compare them
to others in the class. Has everyone constructed exactly the same triangle. Are all 4 cm, 50°, 7 cm
triangles exactly alike?   If you have an idea for a different way to construct this triangle, try it out, and
see how it compares.

III.   ASA Given $\angle$ A = 45°, c = 4 cm, $\angle$ B = 70° (side c is between
the two given angles)
Start a new sketch and once again translate an initial point A , 4 cm
at an angle of 0 degrees, and relabel the point B. Now translate point
A, 45° and 1 cm. Change the straightedge tool to **Ray,** and construct
the ray AA'. Vertex C must lie on this ray. Similarly, construct a ray
starting at point B such that angle B is 70 degrees. Construct the
intersection point of the two rays, and relabel it C. Your sketch should now look like the one on the right.
Hide the rays and the extra points, and connect the three vertices. Now drag on the vertices and edges of
this triangle to convince yourself that it is a rigid construction. Measure the other two sides and the third
angle, and compare your measurements to others in the class. Has everyone constructed exactly the same
triangle. Are all $\angle$ A = 45°, c = 4 cm, $\angle$ B = 70° triangles exactly alike?
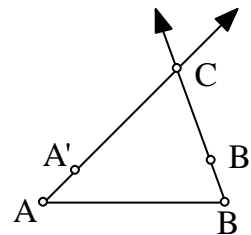
IV. (SSA Ambiguous Case) Given: $\angle$ A = 30°, c = 5, a = 3. Construct two different triangles that have
these conditions, and thus show that SSA is not a condition for congruence. Good Luck

Computer Lab # 21 - SSSS Quadrilateral Investigation

What about constructing a quadrilateral given 4 side lengths? Is there a unique quadrilateral for any 4 given sides? Let's investigate.

1. Construct a quadrilateral ABCD with sides AB = 3, BC = 4, CD = 5, and DA = 6. This construction will be similar to the SSS construction that was done in an earlier activity, but with a slight but important difference. Put down a random point A, and then translate A 3 cm at an angle of 0 degrees. Change A' s label to B. Connect AB. Now translate B, 4 cm in any direction. Construct a circle centered at B going through B'. What is true about all the points on this circle?

_____

Since vertex C must lie somewhere on this circle, use the point tool to put down a floating point on this circle, and re-label it C. Using this technique of translating a specified distance and then constructing a circle, construct circles centered at C with radius 5 cm, and centered at A with radius 6 cm. Display those two circles in different colors, or with thick lines as in the sample sketch below.
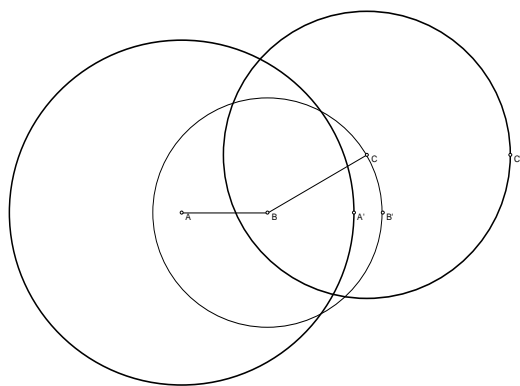
The reason for the different colors or the thick lines for the last two circles, is to distinguish them from the first circle. Where does the vertex D have to be?
If you said at the intersection of the two circles, you are a star. Notice that they intersect in two places. Choose either point of intersection, label it D, and connect the verti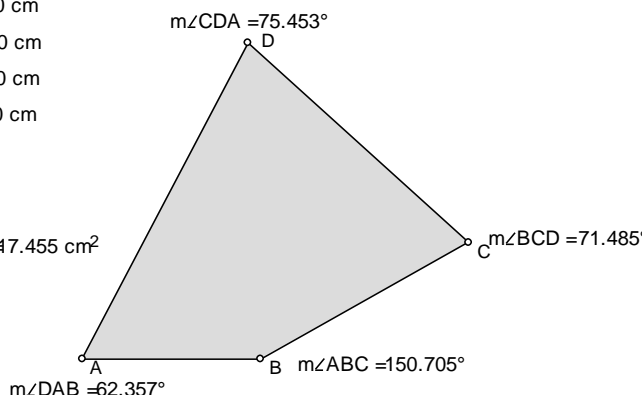ces to form quadrilateral ABCD. Now, form the polygon interior, hide all the construction objects, measure the quadrilateral's area, and also measure the four interior angles. **Remember, C was a floating point on a circle!**

Your sketch should now look like the one to the right

2. Is your quadrilateral unique? _____ If not, try and find the angle measures **to the nearest degree** of the 3-4-5-6 quadrilateral with the largest area. Enter results in the 1st column of the chart on the next page..

3. When you reach this point, see the teacher so that you can be assigned one of the quadrilaterals listed in the chart on the next page. Repeat steps 1 and 2 above with your newly assigned quadrilateral. Record your results on the blackboard, and in the chart.
Assume the lengths given represent the sides AB, BC, CD, DA in that order.

m $\overline{DA}$ =6.000 cm
m $\overline{DC}$ =5.000 cm
m $\overline{BC}$ =4.000 cm
m $\overline{AB}$ =3.000 cm

m∠CDA =75.453°

Area ABCD =17.455 cm²

m∠BCD =71.485'

m∠ABC =150.705°

m∠DAB =62.357°

4. Some questions to ponder

a) What if the sides were in a different order like 3,5,4,6? Are the two maximum area
quadrilaterals with the same side lengths in different order congruent to each other? _____
Do they contain the same area? _____

b) Is there any relationship between the angles that is consistent for each case?

c) Notice that the side lengths in each case above total to the same value of 18. Which one had
the largest area, and why? _____.

d) Is there a quadrilateral with a perimeter of 18 that has a larger area than this one.

Enter the angles for the maximum area quadrilateral. Round off to the nearest degree.
Length of Sides of the quadrilateral

Angle	3-4-5-6	3-5-4-6	3-3-4-8	2-4-7-5	3-5-5-5	4-5-4-5	3-8-5-2
BAD							
ABC							
BCD							
CDA							
Max Area							

5. Write a sentence or two that would summarize your conjecture about maximum area quadrilaterals.

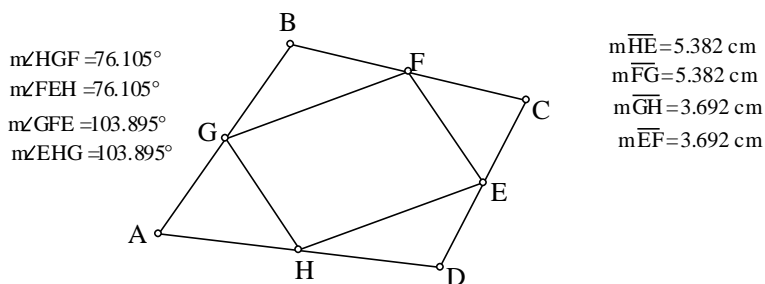# Computer Lab #22 - Midpoint Quadrilateral Investigation

Definition - The *midpoint quadrilateral* is the quadrilateral obtained by connecting the four midpoints of the sides of an initial quadrilateral.
In this activity, you are going to investigate and try and classify the type of *midpoint* quadrilateral that results when the initial quadrilateral is a variety of familiar quadrilaterals. While investigating and identifying these midpoint quadrilaterals, think about why the results are turning out the way they do. What are the underlying reasons?

1. To start, open a new sketch, and using the <u>segment tool</u>, click and drag a circuit of four line segments to construct a generic quadrilateral ABCD. Select all four sides, go to the **Construct** menu, and choose **Midpoint**. Now, choose Segment from the Construct menu and you should have the *midpoint* quadrilateral EFGH inside the *parent* quadrilateral ABCD. While all four segments are still selected, measure their lengths. Also measure the four interior angles of the midpoint quadrilateral EFGH. Choose select all from the **Edit** menu, and then **Create New Tool** from the <u>Custom Tool</u> drop down menu menu, and give it the name *midpointquad*. The custom tool you have now created should have four points as the necessary Given when you click on **Show View Script** from the <u>Custom Tool</u> drop down menu. This is the script you will use in order to construct midpoint quadrilaterals from the four vertices of the parent quadrilateral. It will also produce measurements of side lengths and interior angles, so that you will be able to easily identify and classify the midpoint quadrilateral constructed from the various parent quadrilaterals. Try out your new custom tool *midpointquad* on the quadrilateral EFGH to create its midpoint quadrilateral. Once you have successfully reached this point, see the instructor in order to be assigned one of the quadrilaterals in the table below.

2. Your job now is to construct a generic version of the assigned parent quadrilaterals listed in the table below, to then use the custom tool *midpointquad* to construct the midpoint quadrilateral, and then to make an accurate classification to record in the table below and on the blackboard. Once you have successfully done this, see the instructor to check your work and receive further instructions.

m∠HGF =76.105°
m∠FEH =76.105°
m∠GFE =103.895°
m∠EHG =103.895°

m$\overline{HE}$=5.382 cm
m$\overline{FG}$=5.382 cm
m$\overline{GH}$=3.692 cm
m$\overline{EF}$=3.692 cm

| Parent Quadrilateral	Random	Square	Rhombus	Rectangle	Parallelogram	Isosceles Trapezoid	Kite
Midpoint Quadrilateral							

1. Try and write up what you think are the key ideas surrounding this investigation. You may want to construct the diagonals of the original quadrilateral and examine what role they play.

2. **The Converse Question**: *What can be concluded about the parent quadrilateral if the midpoint quadrilateral is a fixed shape*.? Try constructing a *parent* quadrilateral from a given *midpoint* quadrilateral for the following cases: Write down your conclusions below.
a) Rectangle   b) Rhombus   c) Square

Computer Lab #23 - Investigating Reflections and Mirrors (Part I)

1. Using the segment tool, construct a long segment AB that almost stretches the full diagonal of the sketch window. On one side of the segment, construct a small random triangle CDE as shown below. Mark the segment as a mirror. (Either select the segment and then choose **Mark Mirror** from the **Transform** menu, or double-click on the segment). Select the vertices and edges of the triangle, and then choose **Reflect** from the **Transform** menu. This produces the image triangle C'D'E'. Connect C and C' and construct the intersection point F of CC' with AB. Your sketch should resemble the one to the right.
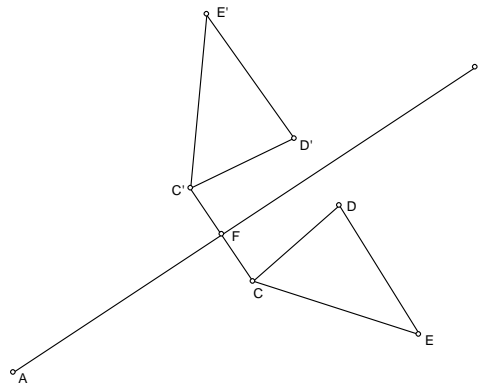
2. Measure the lengths CF and C'F as well as angle CFA. Drag on C as well as endpoints A or B and observe the behavior of the figure as well as your measurements. What seems to always be true about AB's relationship to CC'. _____. Complete the following statement:
If P' is the image point of P after a line reflection has been applied to P, then the reflecting line must be the _____ of segment PP'.
Put a floating point G in your sketch. Find a way to construct where G' will be located after reflecting about AB without using a reflection. Test out your construction idea by reflecting G.

3. If you suspected the two figures at right were congruent and related by a reflection, how could you confirm your guess and reconstruct the reflection line? (i.e. the mirror). Write in the space below.
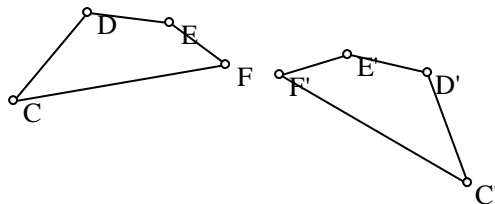
4. What is true about side lengths of the original and image figures? _____.

What is true about the angles of the original and image figures? _____.
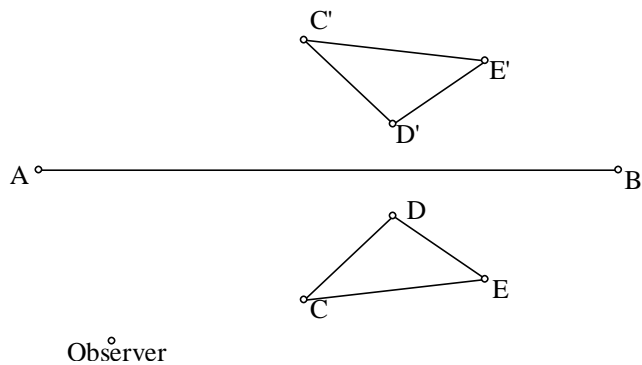A transformation that preserves distance and angle measure is called an *isometry*.
Complete the following: **Any image figure formed by a line reflection will be _____ to the original figure.**

5. Notice the original figure at the top of the page is *oriented* so that CDE are clockwise in that order. Is this true about C'D'E' in the image figure? _____. In general, what will be true about the orientation of a figure relative to its reflection?

What phenomenon do you observe everyday that is an illustration of this characteristic about reflections?

Investigating Reflections and Mirrors (Part II)

1. Start a new sketch. Using the <u>Segment Tool</u>, hold the shift key down, and construct a horizontal segment AB starting in the middle of the screen and extending towards the right side. (When holding the shift key down in Sketchpad, segments are drawn at angles in 15 degree increments.) This will become our mirror. Now construct a random scalene triangle CDE just below segment AB. Using the <u>Point Tool</u>, put a floating point somewhere below the triangle. This point will represent the eye of the observer, so use the text tool to relabel the point "Observer". (Double click on the label in order to change its name.) Mark the segment AB as a mirror by double-clicking with the selection tool, or using the **Transform** menu. Select all the vertices and edges of triangle CDE and choose **Transform/Reflect**. Your sketch should now look like the one to the right. Save this sketch under the name *Mirror* for later use.

2. The object of the following constructions is to show how the path of light rays results in the observer seeing the image triangle. Since there are an infinite number of points in the triangle that the observer is seeing reflected, this construction will work with the single point C to demonstrate the principle. Connect C' and the Observer with a segment, and construct the point of intersection G of that segment and the mirror AB. Now hide the segment connecting C' and Observer. Connect G to Observer, and G to C. Select both of these segments, hold the shift key down, and make these segments dotted and red in color using **Line Style** and **Color** in the **Display** menu. (Holding the shift key down changes the style and color only for those objects selected.) Now construct segment GC', and make it style dotted and color blue. The red dotted lines represent the path of light allowing the observer to see point C. Measure the lengths of Observer to G, GC, and GC'. Measure the two angles formed by the red segments representing the light rays, and the mirror AB. Your sketch should now look like the one to the right.
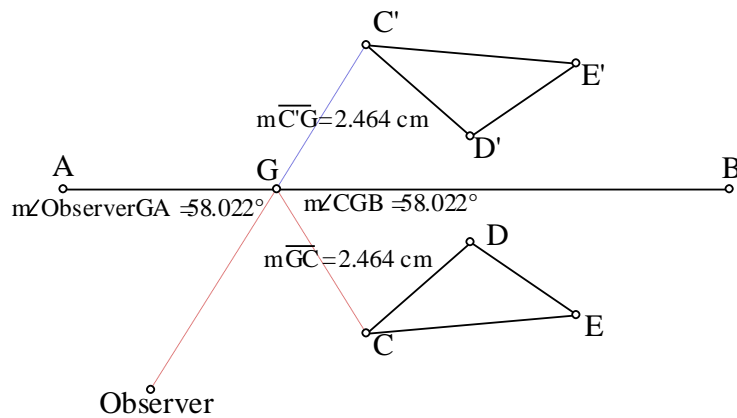
3. Feel free now to move the observer, point C or any part of triangle CDE, or even the mirror AB, and observe what happens. This phenomenon of light ray reflection in order to see the image triangle is of course taking place to every single point in triangle CDE. Notice the important issue of orientation change in the reflected image from the original figure.
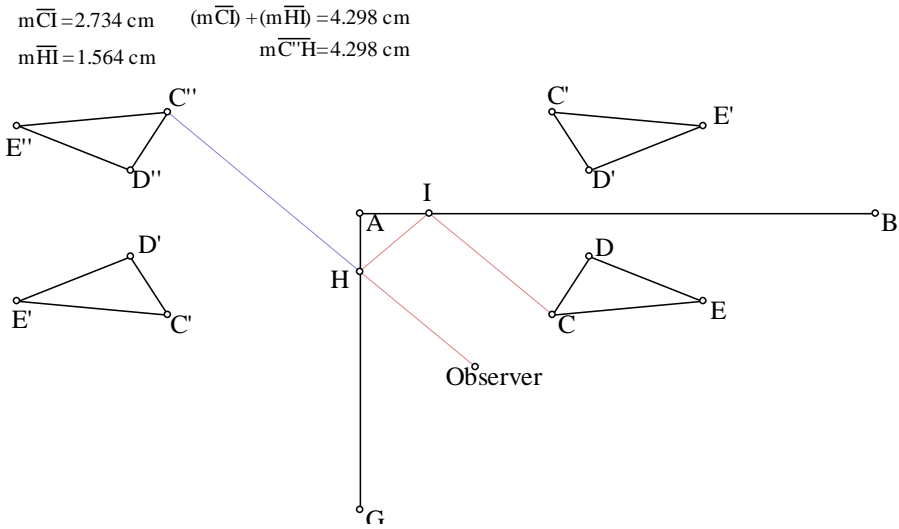
Investigating Reflections and Mirrors (Part III)

1. Open up the saved sketch *Mirror.gsp* from Part 1 of this lab. We are now going to add another mirror that is perpendicular to AB. Hold the shift key down, and using the <u>Segment Tool</u>, click the cursor exactly on point A and then drag down towards the bottom of the sketch window. This new segment, (AG in my sketch to the right), is the second mirror. Mark this segment as a mirror by double-clicking on it, and reflect both triangle CDE, and C'D'E' using this mirror. Your sketch should look something like the one to the right. Note that there are now two triangles labeled C'D'E'.
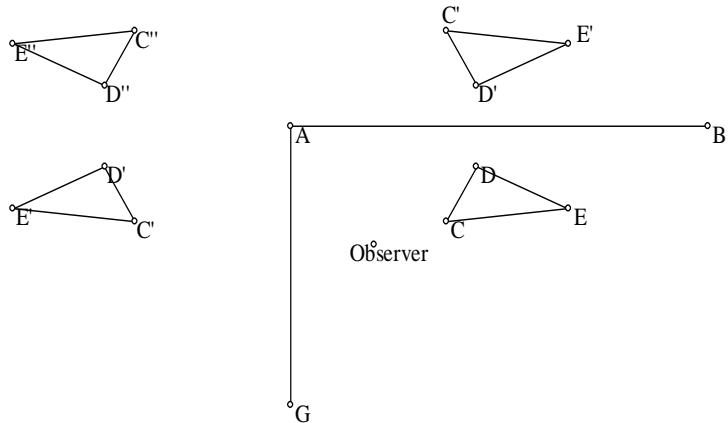
2. Once again, we want to demonstrate the path of light from a single point C that reaches the Observer and causes the viewing of image point C". With the Observer located as shown in the diagram above, construct segment C" to Observer, and the intersection point, (H in the diagram to the right), of that segment with mirror AG. Now hide the segment from Observer to C". Now construct the segment from this intersection point H to point C' in the triangle above mirror AB, and find the intersection point of this new segment with the mirror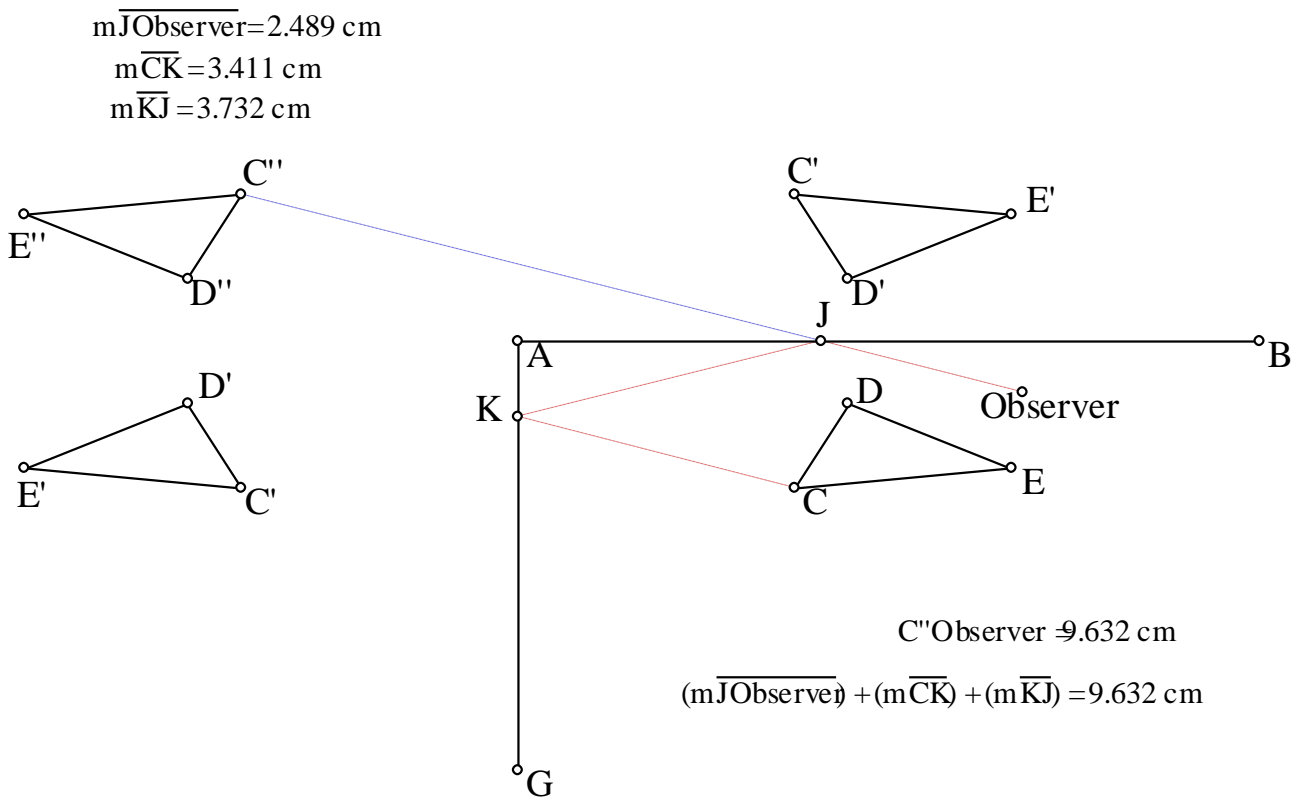 AB, (I in the diagram to the right). Hide segment HC'. Connect C to I, I to H, and H to the Observer. Change the line style of these three segments to dotted, and the color to red. Once again, this represents the path of a light ray. Connect H to C" and make it dotted and blue. Use the calculator to add up the lengths of the three red dotted lines, (i.e. the path of light), and the distance from the observer to the image point C". What do you notice? Move the Observer around. In particular, notice what happens when the Observer crosses over the imaginary line running through C", A, and C. It appears that the light rays disappear because when the Observer is located on the other side of the line, the path of light to see point C initiates off of mirror AG instead of mirror AB. Let's see if we can fix this problem.

$m\overline{CI} = 2.734$ cm $(m\overline{CI}) + (m\overline{HI}) = 4.298$ cm

$m\overline{HI} = 1.564$ cm $m\overline{C''H} = 4.298$ cm

Investigating Reflections and Mirrors (Part III continued)

3. Leaving the Observer above imaginary line AC, connect Observer to C" and construct intersection point J with mirror AB. Now hide the segment. Construct segment JC' using the C' in the image triangle to the left of mirror AG. Construct the intersection point K of this segment with mirror AG, and hide the segment JC'. Connect C to K, K to J, and J to Observer. Change the line style of these three segments to dotted, and their color to red. Once again, this represents the path of a light ray. Connect J to C" and make it dotted and blue. Once again, verify that the distance the light travels from C to the Observer is equal to the distance between the Observer and the image point C". Your final sketch should look like the one below. Now move the Observer around anywhere you want, including inside triangle CDE. Feel free to drag on any part of the diagram including the mirrors.

It is important to notice that the image triangle C"D"E" is oriented the same as the original triangle CDE. This implies that a double reflection is the same as a rotation. Can you see that the image triangle C"D"E" could also be constructed with a 180 degree rotation of triangle CDE about center point A.

$m\overline{JObserver}=2.489$ cm
$m\overline{CK}=3.411$ cm
$m\overline{KJ}=3.732$ cm

C"Observer = 9.632 cm

$(m\overline{JObserver}) + (m\overline{CK}) + (m\overline{KJ}) = 9.632$ cm

Investigating Reflections and Mirrors (Part IV)

What is the shortest length mirror that will allow a person to see their entire body? Where on the wall should the mirror hang? How far away should the person stand from the mirror, or is the distance from mirror not a factor? What about the location of a person's eyes? Does that effect length or location of mirror?

All of these questions can be answered by modeling the problem on the computer and doing some measuring and investigating. The sketch and outline below will set up a very flexible model that should enable you to carry out this investigation.

The model consists of a horizontal line segment AB representing the floor. At the midpoint of AB is the base of a vertical ray representing the wall that has the mirror. The two floating points on the ray represent the top and bottom of the mirror. A floating point on the right side of the mirror and on the segment AB is labeled "Feet". Construct a vertical line through "Feet" and put two floating points on that line which will be the top of the person's head, and their eyes. Hide the line and connect head to feet with a segment. Mark the Mirror/Wall as a mirror from the **Transform** menu, and reflect the person so that the image appears on the other side of the mirror. Connect the head and feet of the image person to the eyes of the real person. Construct the other lines shown for a nice visual effect, and voila' you have a model.
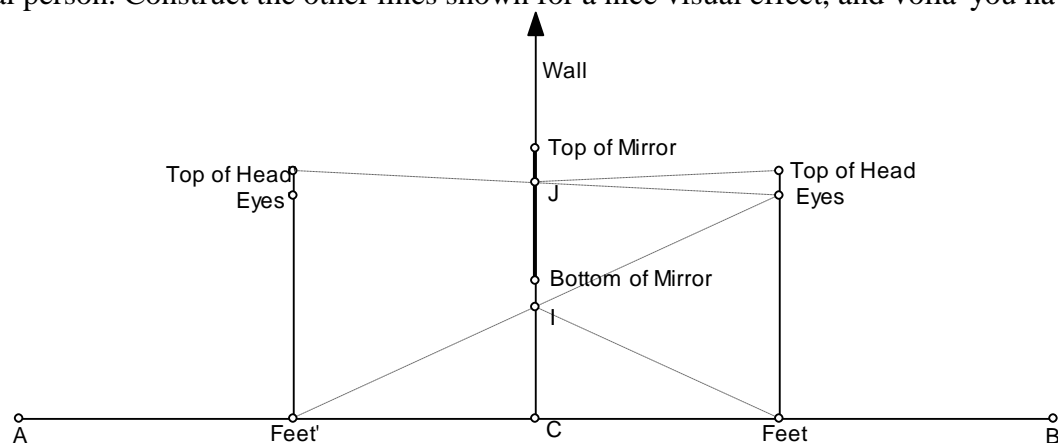

To investigate, you will want to measure the length of the mirror, the height of the person, and anything else you think important to answer the general questions posed at the top of the page, and the specific questions posed below. Good luck

1. How long should a mirror be, so that a 6 ft person standing 6ft away from the mirror will be able to exactly see their entire body? What if the person was 3 ft away, or 12 ft away? What if the person was 7 ft tall? 5 ft tall?

2. Once the optimal size mirror is determined, where should it be placed on the wall?

Computer Lab #24 - Area Investigations
Part I

1. Start a new sketch, and construct a line segment AB. Construct a point C somewhere above the line, and then select C and segment AB and click **Construct/Parallel Line**. Using the Point Tool, put points D and E on this parallel line so that they are near the edges of the viewing window. Hide the line and point C, and construct segment DE. The sketch should now look something like the one at right.
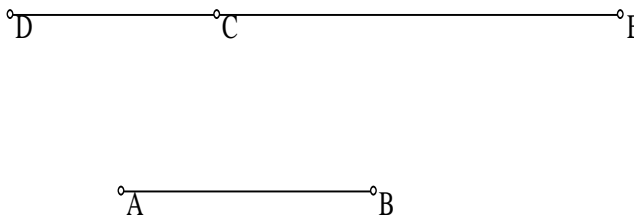
2. Use the Point Tool to put another point F on the segment DE, and then construct triangle FAB. Select points F,A, and B, and then click **Construct Triangle Interior**. While the polygon interior is still selected, measure its area and perimeter. Now the sketch should resemble the one at right.

Area FAB = 13.089 cm^2

Perimeter FAB = 26.069 cm
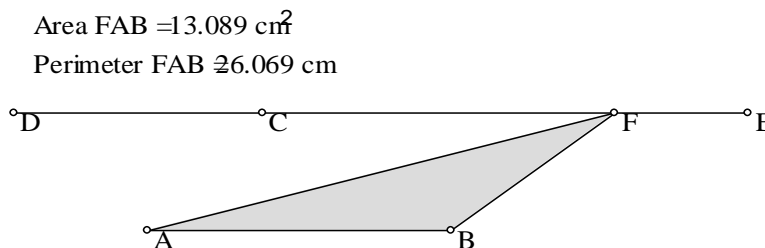
3. Slide F back and forth along segment DE and observe the area and perimeter measurements. Comment below on why you think the area is staying constant despite the perimeter changing.

4. Make an observation about what location of F produces the smallest perimeter.

5. Sketchpad can move F back and forth automatically. Select F and click **Display/Action Buttons/Animation,** choose to animate F bidirectionally along the segment at medium speed and click OK. Click on the animate objects button, and the animation begins. To stop the animation, simply click the animate objects button again. Drag point C to another location. Click the animate button again. By double-clicking on this animate button with the Text Tool, GSP allows you to give it a name in order to distinguish it from other action buttons that you may create in a sketch. Try it.

6. Experiment a little with animation by creating other action buttons that animate at different speeds or directions. Select points F and D before choosing **Edit/Action Buttons/Animation**. Although nothing earth shattering is going on in this particular animation, it does give you an idea of the power of GSP to let the user be creative.

Area Investigations - Part II

1. Start a new sketch. Construct a line segment AB, and use the
Point Tool to put a point C on the line segment AB. Select both C
and the segment AB, and click **Construct/Perpendicular Line**.
Use the Point Tool again, and put a point D on the line. Construct
triangle DAB. Also, select the three vertices D,A, and B and click
Construct/Triangle Interior. Measure the area of the triangle.
Your sketch should look like the one at right.

Area DAB $= 8.890$ cm^2
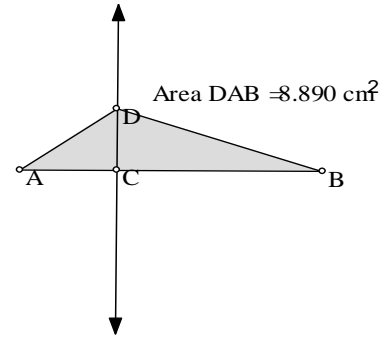
2. Drag D up and down the line perpendicular to AB and observe
the area measurement. The area changes because the height of triangle ABD is changing. What
measurement in the sketch is equivalent to the height of the triangle? _____. Another way to see the
area of the triangle is to calculate it through the formula *Area = .5* base* height.* To display the height,
select points C and D and click **Measure/Distance**. Do the same thing to get the length of the base AB.
Now click **Measure/Calculate** to display the Sketchpad calculator. Drag the calculator to a position on
the screen that allows you to see the measurements of CD and AB. Click on the measure CD and its value
is displayed in the calculator window. Click the asterisk (*) on the calculator for multiplication, then click
the measure of AB. Once again, click the asterisk (*) on the calculator for multiplication, and then click
.5. Click **OK**, and the calculated area should be displayed and be equal to the area measurement already
there. Any of these measurements can be dragged around the sketch window using the Selection Tool.

3. Double-click on point C to mark it as a center.
Select D and then click **Transform/Dilate**. Set
the scale factor to 2 (New) / 1 (Old) if it isn't
already. Click **OK**. The point D' is constructed
so that CD' is twice the length as CD. Measure
CD' to see if this is true. It should remain true
even if you drag D up and down the line. Your
sketch should now look like the one at right.

Area DAB $= 12.364$ cm^2
DC·AB·0.5 $= 12.364$ cm^2

CD' $= 5.646$ cm
DC $= 2.823$ cm

AB $= 8.759$ cm
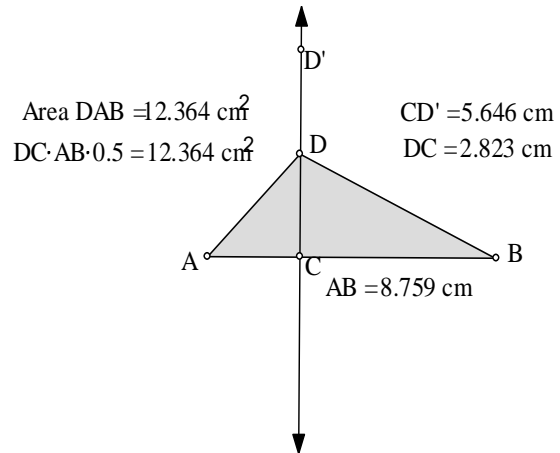
4. Construct the triangle D'AB and its interior.
Before measuring, try and conjecture about how
this area will compare to the area of triangle
DAB. Think carefully about what lengths are
involved in the area calculation. Measure the area of D'AB to check if your conjecture is right. Drag on
points C or D to see if the basic relationship holds. How would the area of D'AB compare to the area of
DAB if D' were dilated so that CD' = 3* CD? Try it.

5. Summary - In a triangle, if the base remains constant, and the height is changed by a **factor** of *f*, then
the area is also changed by a **factor** of ___.

Area Investigations - Part III

1. Start a new sketch. Start with a floating point A, and some random triangle BCD that is nearby. Form the triangle interior of BCD. Select A and choose **Transform/Mark Center**. Select the polygon by simply clicking anywhere inside, and then choose **Transform/Dilate**. Respond to the dialogue box with a ratio of 2:1 for New to Old. A new triangle is constructed that is *similar* to the first. Write down what must be true about these triangles in order for us to say they are *similar.*

_____

Verify that this is the case. Your sketch should look like the one at right. The reason the vertices of the dilated polygon don't show is because you dilated the polygon interior only. If you want the vertices also, they must be selected as well before dilating.
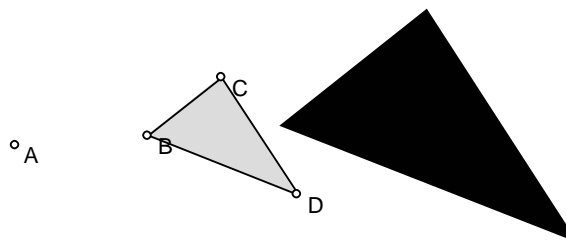
 To investigate perimeter and area, select both polygons and measure both perimeter and area for each one. Then select these measurements and go to the **Calculator** under the **Measure** menu, and display the ratio of their perimeters and their areas. Repeat the dilation and measuring using the different dilation factors in the table below.

Dilation Factor of New Triangle	Ratio of Perimeters New:Old	Ratio of Areas New:Old
2		
3		
4		
.5		

If the diagram gets too messy, simply hide or cut any figure and measurements you are finished with. To be consistent, let's always divide the dilated polygon's measure by the original's. Click and drag A, B, and triangle BCD and observe what happens to both the picture and your measurements. Record your results in the table. Keep your original triangle BCD reasonably small. You might want to change colors for your different polygons for a nice visual effect.

4. When you have completed the chart, write down a clear sentence stating your conjecture about the relationship between similar triangles and their perimeter and area.

5. What if the original figure was not a triangle. Repeat part of the investigation using another figure instead of triangle BCD. Use another polygon, a circle, or a sector of a circle, (ask me and I'll show you how to construct a circular sector). Does your conjecture appear to be true for similar figures other than triangles?

# Computer Lab #25 – Pythagorean Theorem

1. Start a New Sketch and construct a random line segment AB. Construct a perpendicular line through A. Put a floating point C on the line. Hide the line. Connect the points ABC and you should have a generic right triangle. Drag on the vertices to test it out. Click **File/Document Options**. A dialogue box opens up that enables you to break this sketch up into separate pages that are easy to access. Replace the 1 in the top box under Page Name with the name *right tri* . Now, in this same dialogue box, click **Add Page/Duplicate** and choose *right tri* when the drop down menu appears. Change the name under Page Name to *pyth1* and click OK. This creates a duplicate page to the one you just created named *pyth1*. Look in the lower left hand corner of your sketch, and you will see two tabs with the names *right tri* and *pyth1*. Multiple pages in a sketch are created this way, and the tabs are used to move back and forth among these pages.

2. You have already created and saved the custom tool *square maker* from previous work on page 10. If you have not worked with custom tools on page 10 yet, go back and do that page before continuing with this lab. This tool should be available by clicking on the Custom Tool icon in the tool box and choosing square maker from the drop down menu. As you may recall, this tool requires two points as the *Given* in order to construct a square. Using this tool, click and drag from one vertex of right triangle ABC to another, and create a sketch similar to the one at right. Be careful in the direction you choose the vertices so that the squares are attached to the outside of the triangle. Simply use the **Edit/Undo** feature if you make a mistake. Use the calculator under the **Measure** menu to verify that the larger square's area equals the sum of the other two smaller areas. Drag vertices around to change the triangle. Explain why this is an illustration of the Pythagorean Theorem:

Area ACA'C' =4.053 cm^2

Area CBC'B' =5.130 cm^2

Area BAB'A' =4.078 cm^2
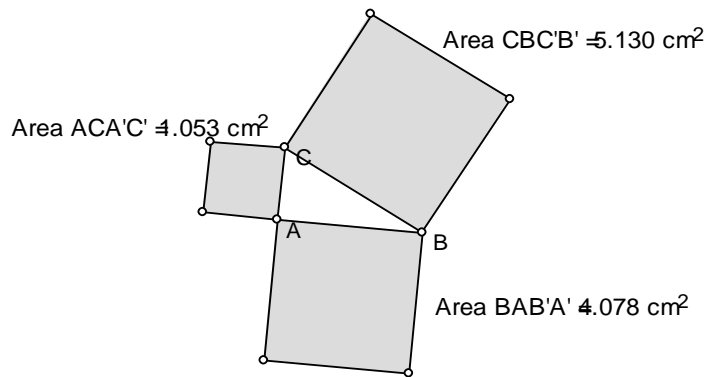
Area - Part IV Pythagorean Theorem (continued)

Open a New Sketch. Draw an arbitrary segment AB with the Segment Tool. Now construct a circle with this segment AB as a diameter. Now, select point A, the circle, and point B. Click **Construct/Arc on Circle**. Hide the circle. Now select the arc only and choose **Construct/Arc Interior/Arc Sector**. The interior of the arc is constructed in a similar manner to the way polygon interiors are constructed. While the interior is selected, click **Measure/Area**. When complete, the sketch should look like the one to the right. Choose **Edit/Select All**, and then create a new custom tool by clicking and holding on the Custom Tool icon in the tool box. Call this new tool *arc maker*, similar to *square maker*. View the script for this tool, and you will notice that it requires two points just like *square maker*. Click Save As and save this sketch in the Tool Folder as *arc maker*. Keep this sketch open for the time being to make sure the tool is available.

Area p1 =21.513 cm^2
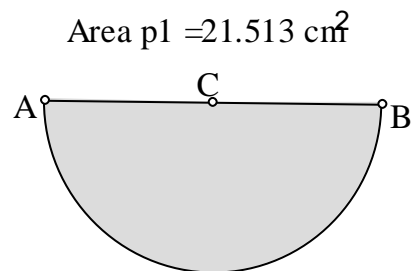
4. Go back to the original window in which you constructed the right triangle and click on the tab in the lower left corner of the window that says *right tri*. This activates the page with just a right triangle on it. As we have done earlier in this lab, **Click File/Document Options**, and make a duplicate page to *right tri*, and call it pyth2. There should now be three tabs in the lower left corner called *right tri, pyth1*, and *pyth2*..

5. Use the custom tool you just created and saved called *arc* to attach half-circles to the sides of the right triangle ABC in your current sketch. Your sketch should look like the one at right when finsihed. As in previous case with squares, test whether the area of the largest half-circle equals the sum of the areas of the two smaller ones. Does this result surprise you?

Area p2 =3.027 cm²   Area p1 =4.497 cm²

Area p3 =1.469 cm²
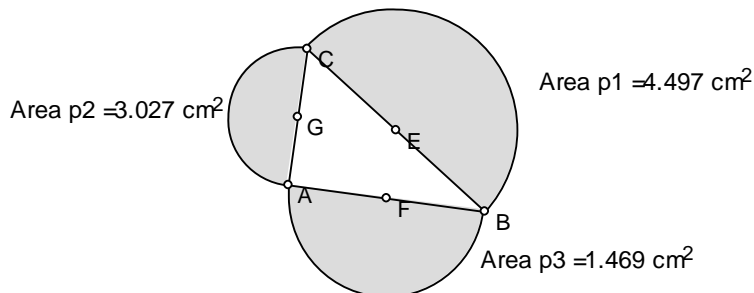
Challenge: Try and construct the figure shown to the right using another new custom tool that you will need to create. Can you make a conjecture based on the examples you have seen in this investigation about what types of figures you could attach to the sides of the right triangle and maintain the area relationship that illustrates the Pythagorean Theorem.

Try and state as clearly as possible in either words or mathematics what all these areas have in common that make any of them useful in illustrating the Pythagorean Theorem.
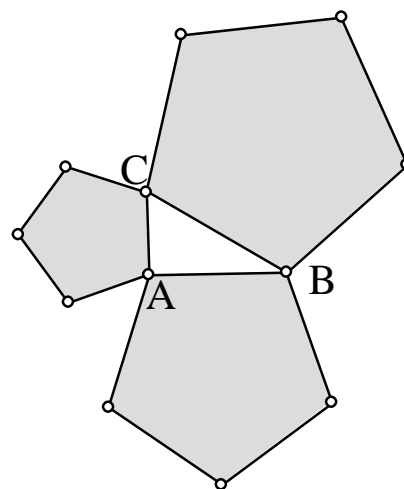
Computer Lab #26 - Coordinate Geometry Questions

1. Start a new sketch and click **Graph/Show Grid**. The coordinate axis along with the origin point and the unit point at (1,0) are displayed, along with graph paper grid lines. Click **Graph**, and choose **Snap Points**. This means that when a point is dragged in the sketch, GSP will automatically snap or move the point to the nearest grid or lattice point location. Let's try it.
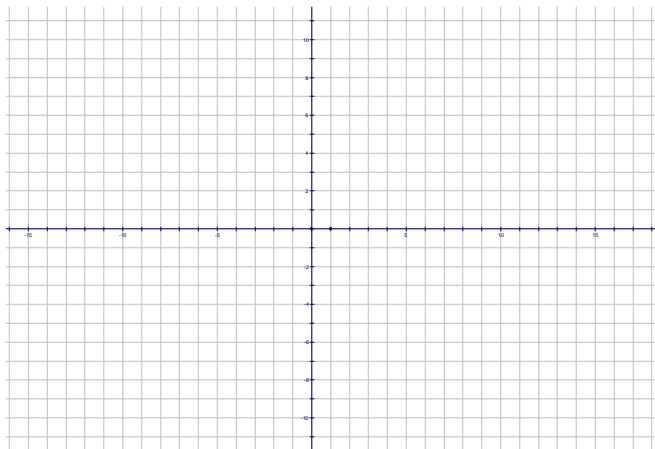
2. Use the <u>Point Tool</u>, and put down 5 or 6 random points in the sketch window that are not on grid lines. Notice that no matter where you try and drag the points, the points end up at grid locations. Use the <u>Selection Tool</u> and drag on any of the points you just constructed. They will only move to lattice locations. Select the points in clockwise or counterclockwise order, and click **Construct/Polygon Interior**. Click **Measure/Area**. Drag any of the points to new locations and notice something curious about the area. It is always an integer, or half an integer. Try and come up with an explanation for this phenomenon.

3. If you turn the Snap to Grid feature off, then the points are free floating as usual. This feature can be turned back on again, but it will only effect points that are dragged, or newly constructed points. Close and don't save the sketch.

4. A lattice point area question. What is the radius of the largest circle that surrounds **exactly** one lattice point? two lattice points? Challenge: three lattice points? *Points on a circle are **not** surrounded by the circle.*

5. Construct a line that passes through the origin, but does not contain any other lattice points. Think carefully about the nature of the slope of this line, and how to construct such an example.

6. ABCD is a square. If A = (0,0) and B = (3,1), what are the coordinates of C and D?

7. Using strictly translations, construct the hexagon pictured to the right that is equilateral but not equiangular. All the vertices are all lattice points.
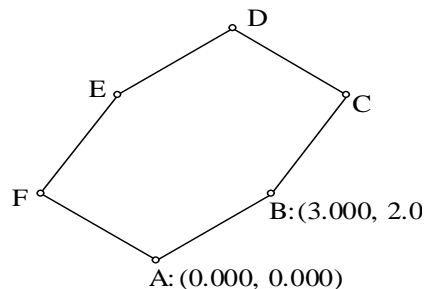
Computer Lab #27 - Data Analysis and Statistics

A. Some preliminary definitions:

Residual- Given a line $(mx+b)$ and a point (x_1, y_1) not on the line, the difference $y_1 - (mx_1 + b)$ is called a *residual*. Its magnitude is the vertical distance between the point and the line. Its sign indicates whether the point is above or below the line.

$y_C - y_D = 3.079$
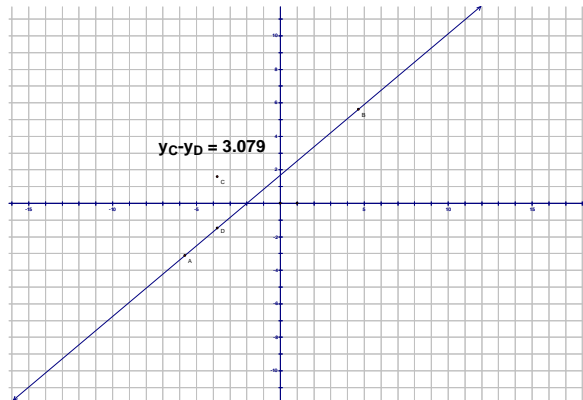
Zero-residual-line - If three data points are not exactly collinear, but they are suspected of conforming to a linear model, then one possible way of fitting a line to the data is the *Zero-residual-line*. This line is parallel to the line connecting the leftmost and rightmost data points, and it makes the sum of the three residuals zero. Unless the three points are collinear, none of the three points is actually on the zero-residual-line.

Median-Median point- For a given set of data points, the *Median-Median point* is the point whose *x*-coordinate is the median of the *x*-coordinates of all the data points, and whose *y*-coordinate is the median of the *y*-coordinates.

Median-Median Line - This is a method of fitting a line to many data points that are not exactly linear, but are suspected of conforming to a linear model. When given many data points, they are to seperated into three different groups of equal size by working left to right. If the total number of data points is not divisible by three, then the three groups should be formed so they are symmetric in number. After forming the three groups, the *median-median point* is to be used as a summary point for each group. Using these three summary points, calculate the *zero-residual line*. This line is called the *Median-Median line*.

Centroid of data points - This point has an *x*-coordinate that is the average of the *x*-coordinates of all the data points, and similarly the *y*-coordinate is the average of the *y*-coordinates of the data points.

Least Squares Regression Line or Linear Regression Line - As with the *Median-Median Line* described above, this is a method of fitting a line to many data points that are not exactly linear, but are suspected of conforming to a linear model. This line passes through the *centroid* of the data points, and has a slope that minimizes the sum of the squares of the *residuals* for all the data points.

B. Creating a Residual Custom Tool

1. Start a new sketch, and change the straight edge tool to the line icon, use the line tool to construct a random line AB, and then use the point tool to put a point C somewhere not on the line. Select point C in the sketch window, click **Transform/Translate**, and translate C one unit vertically up or down to create point C'.

2. Construct the vertical line through C and C'. Construct the intersection point D between this line, and the original line AB. Now select points C and D and choose **Measure/Ordinate (y)**. A coordinate system and axis will automatically appear, and the y-coordinate for these two points in the sketch window will be displayed. Now, choose **Measure/Calculator**, and subtract the *y*-coordinate of point D from the y-coordinate of point C. The value of $Y_C - Y_D$ will appear in your sketch window, and this is of course the residual of point C relative to line AB.
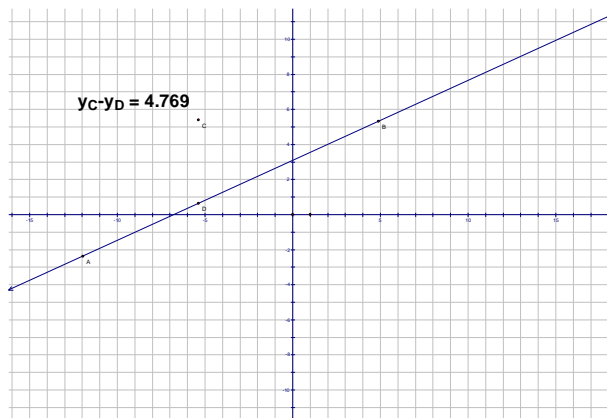
3. Select point C', line CC', and the ordinate measures of points C and D, and click **Display/Hide Objects**. Your sketch should look like the one at right.

4. Click on point C and drag it around. Notice the changes, especially as you drag the point above, below, and on the line AB. Now lets make a residual custom tool. In the sketch, select point C, the measure $Y_C - Y_D$, and the line.

Click and hold on the <u>Custom Tool</u> in the tool box, and choose **Create New Tool**. Give it the name *residual* when prompted. Click and hold on the <u>Custom Tool</u> in the tool box, and choose **Show Script View**. Notice the necessary *Givens* for this new custom tool are a point and a line. Click and hold on the <u>Custom Tool</u> in the tool box, and select this new tool *residual*. In the sketch, click somewhere off the line, and then click on the line itself. Notice you have just repeated all the steps necessary in just those two clicks in order to see the residual of a new point with respect to that line.

C. Creating a Residual Squared Script

1. Clearly we can easily get the square of a residual by just selecting the residual measurement and then squaring it using the calculator capability of GSP. However, we are going to do create a residual squared custom tool instead. This will be slightly more work, but will produce a nice visual effect.

2. Double-click on D to mark it as a center, and rotate C 90 degrees to C'. Now, double-click on C', and rotate D 90 degrees to D'. DCD'C' should form a square. Select these 4 points in order and choose **Construct/Quadrilateral Interior**. Choose **Measure Calculate**, and square the residual $Y_C - Y_D$. We now have a visual for the residual squared, and its actual value. To make things a little less cluttery, select points C', D', and the measure of $Y_C - Y_D$, and hide them. In the sketch, select point C, the measure $(Y_C - Y_D)^2$, the colored square interior, and the line. Click and hold on the <u>Custom Tool</u> in the tool box, and choose **Create New Tool**. Give it the name *residsquared* when prompted. As in step 4 above, take a look at the script by choosing **Show Script View** after clicking and holding on the <u>Custom Tool</u> icon. As in the *resid* custom tool, the necessary *Givens* for this new custom tool are a point and a line. Choose this new custom tool, and test it out by clicking off the line first, and then the line itself. Notice you have just repeated all the steps necessary in just those two clicks in order to see the residual squared of a new point with respect to that line. Save this sketch in the Tool Folder so that these custom tools will be available to you anytime. Do not close this sketch yet.

D) *Zero-Residual Line* **Lab**

1. Start a new sketch, and choose **Graph/Show Grid**. Using the point tool, place three points on the grid such that A = (−8–5), B = (1,–1), and C = (4,3). Calculate the centroid of this data (the average of the x-coordinates and the average of the y-coordinates) and you should get the point (−1,–1). Put the point D = (−1,–1) on the grid also. Hide origin A and unit point B to make diagram less cluttery, and also choose **Graph/Hide Grid**. Now, place the cursor for the line tool on top of point D and click and drag to open space form a line DE. Notice you can drag on point E to change the slope of the line. Drag E so that your line looks like a good linear fit to the three data points A, B, and C. Do not drag on any other point at this time.

2. Choose the resid tool from the custom tool box, and select A and the line. Repeat for points B and C, so that the three residuals are displayed as shown in the figure at right. Choose **Measure/Calculate** to calculate the sum of the three residuals. Are you surprised at all with the result? If your sum of the residuals is exactly 0.0, it would appear you have stumbled upon the *zero-residual line* for this data.

$y_A\text{-}y_F = 0.477$
$y_B\text{-}y_G = \text{-}1.279$
$y_C\text{-}y_H = 0.802$
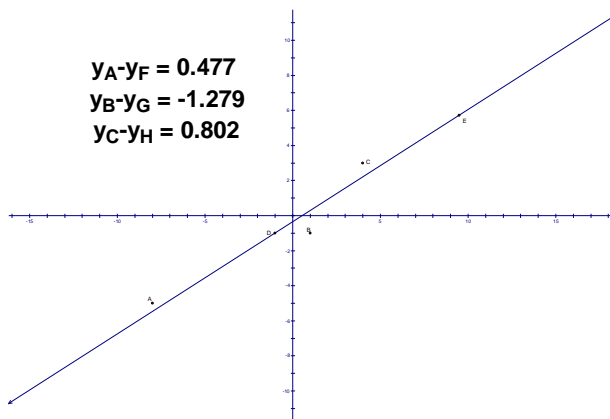
3. Click and drag on point E so that the slope of your line changes. Do you notice anything interesting about the sum of the residuals as your line changes? As you may have noticed, it turns out that any line through the centroid of three data points is a *zero-residual* line. Therefore, there are infinitely many. To find the unique *Median-Median Line* for this data, you need the *zero-residual* line with the same slope as AC. Without any further instructions, see if you can construct this line and display its exact equation. Write the equation of the *Median-Median Line* in the space below.

4. To verify that you have indeed found the *Median-Median Line* for this data, let's check it out with the calculator. Using the TI-82 or 83, enter the *x*-coordinates of the three data points into L1, and the three *y*-coordinates into L2. Then choose **Stat/Calc,** and pick option 4 which is Med-Med. Enter in L1, L2 after the words Med-Med appear on the screen, and the values $\begin{array}{l} a = .66\overline{6} \\ b = -.33\overline{3} \end{array}$ should appear on your screen. Hopefully this agrees with what you found on the computer.
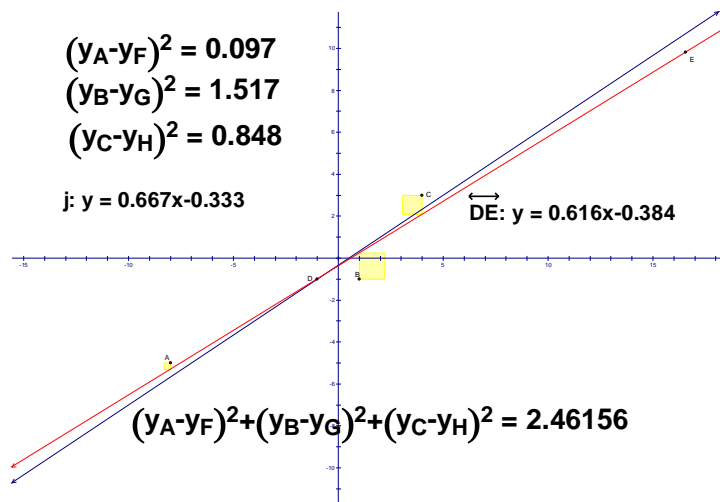
E) **Least-Squares / Linear Regression Line Lab**

1. If you have just completed the previous activity, hide everything in the sketch except original data points A, B, C, centroid D, line DE, the *median-median* line you just constructed, and its equation. Change the color of the *median-median* line to red so as to distinguish it from line DE.

2. Using the *residsquared* tool from the custom tool box, select point A and the line DE in order to calculate the residual squared and to see the visual square created as well. Repeat this for points B and C. Use the calculator to find the sum of the three squares of the residuals. Clearly, this is some positive number that will change as the slope of line DE is changed by dragging on E. Move E so as to minimize the sum of the squares of the residuals. You may want to change the accuracy displayed for this sum od residuals squared by double-clicking on it with the <u>Text Tool</u>, and then using the Value tab to change accuracy displayed.

3. Once again, let's check the computer solution by comparing it to the solution on the calculator. Since the data points have already been entered into L1 and L2 of the calculator, simply choose **Stat/Calc** and option 5 for LinReg (ax + b). When LinReg (ax + b) appears on the home screen, enter L1,L2 and hit enter. How do the calculator's values of a = .615 and b = -.385 compare to your sketchpad solution? Your final sketchpad sketch should resemble the one shown to the right.

$(y_A\text{-}y_F)^2 = 0.097$
$(y_B\text{-}y_G)^2 = 1.517$
$(y_C\text{-}y_H)^2 = 0.848$

j: y = 0.667x-0.333

$\overleftrightarrow{DE}$: y = 0.616x-0.384

$(y_A\text{-}y_F)^2+(y_B\text{-}y_G)^2+(y_C\text{-}y_H)^2 = 2.46156$

So what is the better way of fitting a line to data that is suspected of conforming to a linear model? Will the two models always be reasonably close as in the example above? There is no simple answer to the first question, but it is safe to say that statisticians rely on linear regression as the method of choice almost exclusively for large data sets. One of the reasons for this is it is relatively easy to compute, and the answer can be given in terms of the means and standard deviations of the two variables and their correlation. However, it should be noted that the linear regression method is much more influenced by *outliers* in a data set than the median-median approach, and thus the two methods can in fact give very different results.
(*Outliers* are data points that fall way outside the overall trend or pattern to a data set) A problem on the next page illustrates this point.

Listed below is some data for you to enter into your calculator and to apply both the *linear regression* and *median-median* methods. Hopefully this exercise will help illustrate this last point.

A sample of car ages in years, and the mileage displayed on the odometers.

Age in Years	1	1	2	3	3	3	4	5	7
Mileage in Thousands	12	14	26	30	33	35	51	60	18

1. Enter the data into L1 and L2 of your calculator, and do a scatter plot of the data points.

2. Calculate the linear regression line and the median-median line for this data, and enter those lines into y_1 and y_2 of the calculator. Notice the difference in their equations. Explain why this would be expected?

3. Calculate the predicted mileage that each linear model would yield for a car that is seven years old.

4. Change the last data point from (7,18) to (7,48). Recompute the linear regression line and the median-median line and notice what has happened. How have the predictions for the mileage of the seven year old car changed? Explain why this has happened?

Computer Lab #28 - 3-D and Perspective Drawing

Since we live in a 3-D world, everything we see has depth, height, and width. We see in *perspective*, so that the farther away an object, the smaller it appears.

Part I

1. The first exercise is to construct a box in *one-point perspective*, which means that there is a single *vanishing point*. Start a new sketch, and construct a rectangle that has a flexible length and width. (There are several ways to do this, but remember to do the construction so that it will stay a rectangle even after dragging on any part of it.)

Construct a point above and to the right of the rectangle, and label it *vanishing point*. Now connect the four vertices of the rectangle to the vanishing point. Your sketch should look like the one at right.
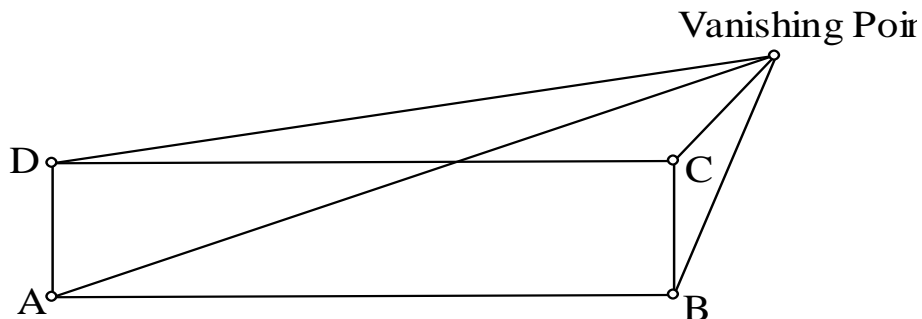
Vanishing Point

2. Construct a floating point on the segment from C to the *vanishing point*. This point will control the depth of the box. Construct lines parallel to AB and BC through this point and then construct two more of the vertices of this box. The last vertex should be easy to construct as well. Hide all construction lines, and then connect the necessary vertices to construct the box. Your sketch should look like the one at right.
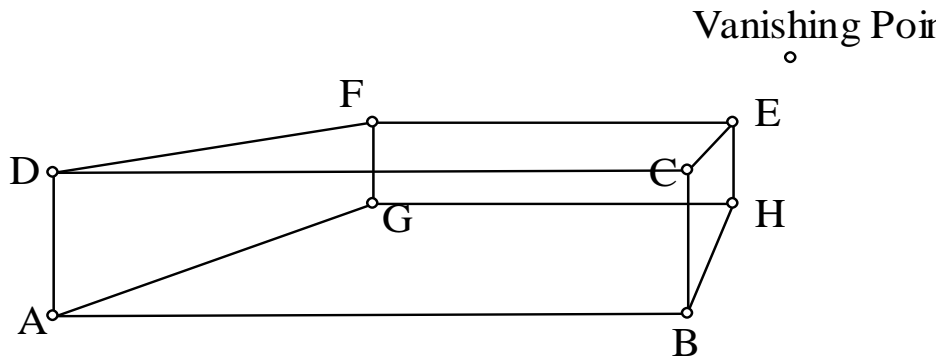
Vanishing Point

3. Now, have some fun experimenting with dragging on the different vertices of the box, as well as moving the vanishing point to different locations. A few questions to ponder:

What must the position of the box be in relation to the vanishing point so that you can:

a) see the top of the box?   b) see the bottom of the box   c) see the left side of the box.
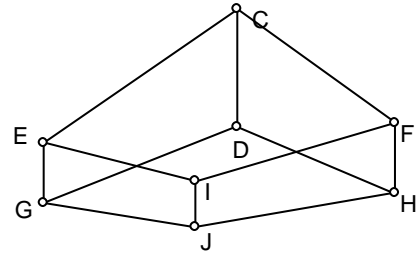
3-D and Perspective Drawing - Part II

The second exercise is to make a drawing of a box in *two-point perspective.* This means that there will be **two *vanishing points*** where the parallel lines of the box recede or converge to these two points.

1. Start a new sketch, and construct an approximately horizontal line segment AB in the middle of the sketch window. Construct a point C below AB and construct a line through C perpendicular to AB. Put another point D on this line below C. Hide the line and connect C and D. Now construct segments CA, CB, DA, and DB. Put a floating point E on segment CA, and floating point F on segment CB. Now draw lines through E and F that are parallel to CD. Construct the intersection points G and H of these lines with DA and DB respectively, then hide the lines. Construct segments EG, FH, AF, and BE. Construct the intersection point I of segments AF and BE. Your sketch should now look like the one below.
Click and drag on C, D, E, and F and observe the behavior of your drawing. Although the top of this box is really a rectangle, verify through appropriate measurements that the quadrilateral EIFC that represents our perspective view of the top of the box is itself not actually a rectangle.
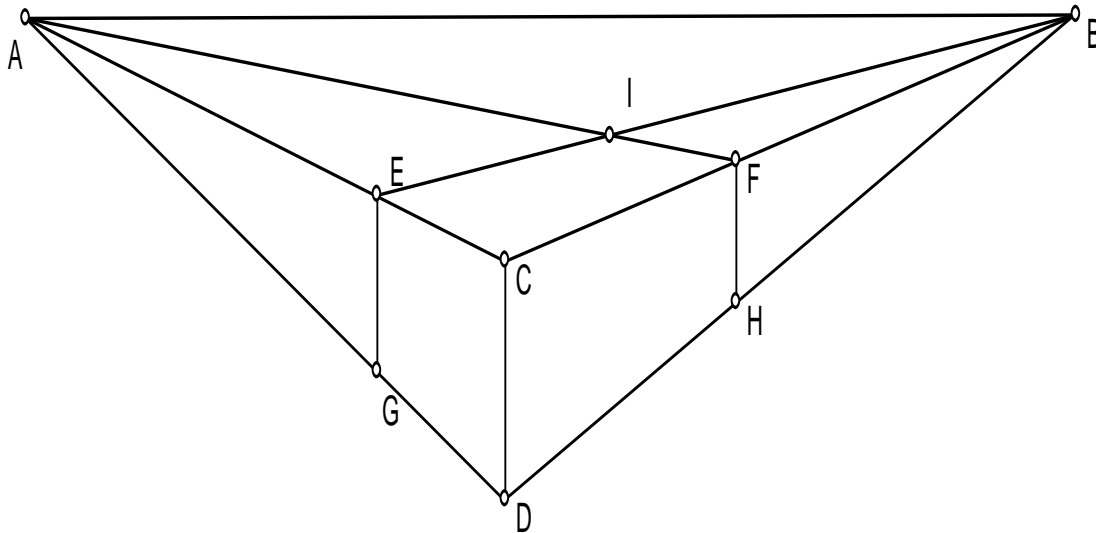
2. Locate the middle point of the side face CFHD, and use this point to find where the midpoints of CF and DH would be located. Although the distances represented are equal, the actual distances in the diagram will not be.

3. Drag CD above the horizon line AB. Add lines to the sketch so that the box is complete. When finished, hide all lines and objects other than the box itself, and the two vanishing points. Now do some exploring by dragging on the vanishing points as well as other objects in your sketch. An example of the box above the horizon is shown at right. Notice that some lines are in view that would be normally blocked from view by the bottom GDHJ and sides of the box.

Which lines in the sketch are actually parallel, as opposed to looking parallel?

Part III - Railroad Tracks

1. Start a new sketch. In this sketch, try and apply what you know about perspective drawing to draw railroad tracks receding to a single vanishing point on the horizon. The tricky part here, is to construct the segments running across the tracks so that they appear equally spaced apart, and will remain that way after dragging on any point. Turn the autoshow label option for points off by choosing **Preferences** from the **Edit** menu.
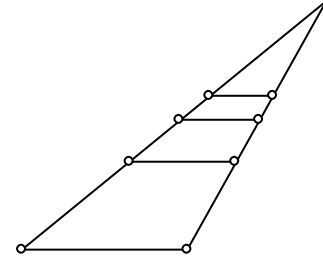
2. If everything goes well, and you have some time, try and construct evenly spaced telephone poles that run along side your tracks, but are offset a short distance. The sketch below shows the first two of these poles. Try and add to some more.
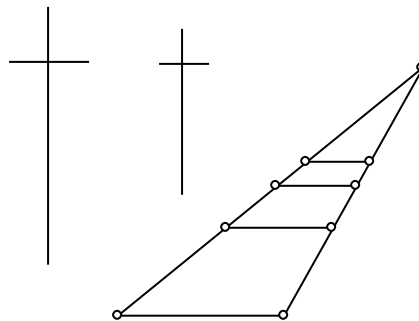
Computer Lab #29 – Conic Constructions - Parabolas

1. Start a new sketch. Using the <u>Segment Tool</u>, construct a segment AB that stretches across the width of the sketch window, and is located about one third of the way from the bottom. Use the <u>Point Tool</u> to construct a point C that is about one inch above the segment AB, and roughly in the middle of the window. Now put the cursor of the <u>Point Tool</u> on the segment AB and click. This constructs a point D that will slide back and forth along AB. Use the <u>Selection Tool</u>, to drag D back and forth. Your sketch should look like the one at right.
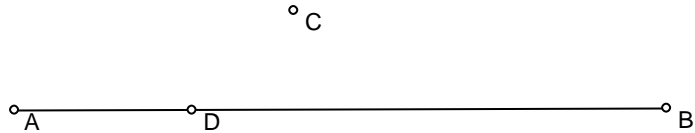
2. Use the <u>Segment Tool</u> to construct segment CD. While it is still selected, click **Construct/Midpoint** to construct midpoint E. Then, use the <u>Selection Tool</u> to select segment CD in addition to midpoint E. Click **Construct/Perpendicular Line** in order to construct the perpendicular bisector of segment CD. What is true about all the points on this perpendicular bisector of CD?

3. Select point D and segment AB and click **Construct/Perpendicular Line** again, in order to construct the perpendicular line to AB passing through D. While this line is still selected, also select the perpendicular bisector of CD, and then click **Construct/Intersection**. This will construct the intersection point F of the two lines. See right. What special property does point F have? Construct FD and FC and measure their lengths.
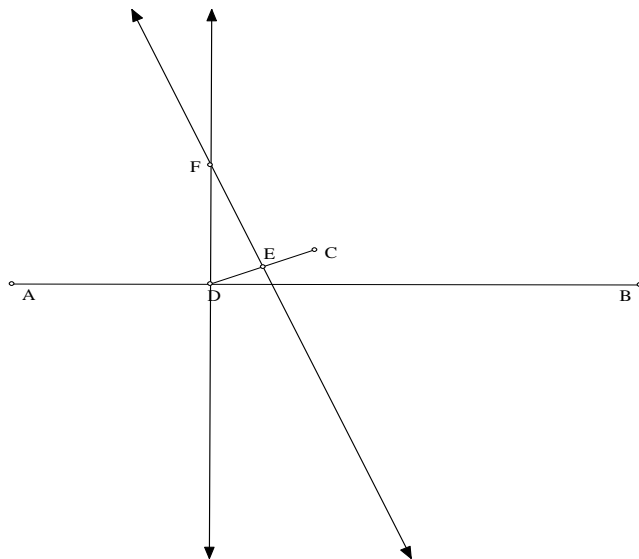
4. Using the <u>Selection Tool</u>, drag D back and forth along the segment AB, observe the path of point F as you do so. To see a trace of the points that F passes through, select F, and then click **Display/Trace Intersection**. Now drag D again, but do it slowly for a better trace. The curve traced out by these points is called a *parabola*.

5. Rather than dragging D back and forth, Sketchpad has animation capability. Select D only, then click **Edit/Action Buttons/Animation**. A dialogue box will appear giving you some options for the animation. Try medium speed at first to see how fast it goes. After clicking OK, An Animate Point button is created that will start the animation when it is clicked. Try it. To stop the animation, click the Animate Point button again. To give the Animate Point button a different name that will allow you to remember what it does, use the Text Tool to double-click on it and then change its name. Change the diagram by moving C to a different location, and then start the animation again. To erase the traces from the screen, hit control-B on the keyboard, even while the animation is in progress.

Parabolas (continued)

6. To construct more permanent curve, also called the *locus* of point F, select point D and point F, and click **Construct/Locus**. The parabolic curve is now permanently constructed. Your sketch should now look like the one to the right.
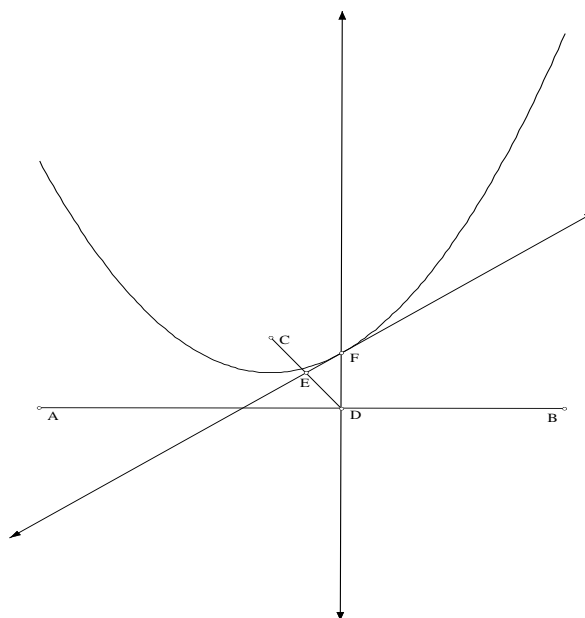
7. Select point F and click **Display/ Trace Intersection** to toggle off the tracing feature on F. Drag point D back and forth and again observe the behavior of the various elements in the construction. To make the diagram less cluttered, select segment CD, the perpendicular line through D, and point E, and click **Display/Hide Objects**. Now, drag point C (called the *focus*), and notice the changes that take place. Make sure you drag C below the segment AB. You should also drag the endpoints of segment AB (called the *directrix*), and observe the behavior of the curve. Summarize your observations in the space below.

8. To sketch the specific parabola whose *directrix* is $y = -1$, and whose *focus* is at (0,1), first click **Graph/Show Grid** to display the coordinate system**.** Also, activate the **Snap Points** feature from the **Graph** menu. It should now be easy to drag points A and B to locations on the line $y = -1$, and to drag the *focus* C to the location (0,1). Sketchpad cannot display equations of curves other than lines and circles, but it is capable of displaying a table of (x,y) values of points on the curve. See #9 below.
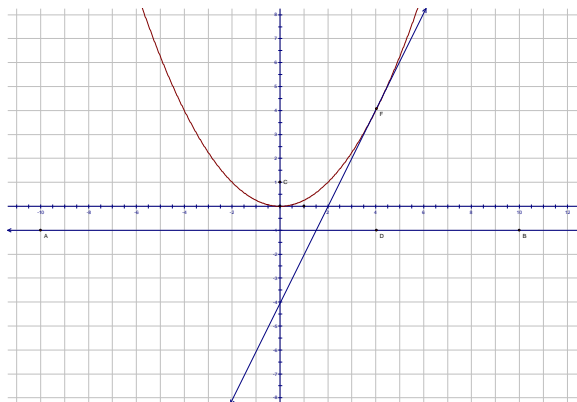
9. To formulate a table of coordinates for points on this parabola, select point F and click **Measure/Coordinates**. Now select the coordinates of point F and click **Graph/Tabulate**. A table is created and the first entry in the table is displayed. Double-click on the table to record that entry permanently in the table. Drag point D and thus point F to a different position. Double-click on the table again, and this new data point is now added to the table. Continue this process until you have about six or seven data points.

Parabolas (continued)

10. Since GSP is limited to displaying the equations of lines and circles only, if we want an actual equation for the parabola with focus at (0,1) and directrix $y = -1$, then we need another way. Enter the values from the table into a graphing calculator. Include (0,0) as well, since this point is clearly on the curve. Do a **QuadReg** on the data, and you should get an equation displayed that is very close to the actual equation of $y = .25x^2$. Now move the focus to (0,2), and the directrix to $y = -2$? Create a new table and determine the regression equation using the calculator. What do you notice happened to the equation. Repeat this process for a couple of more examples and try and predict what is going on with the coefficient of the *x*-squared term in the equation, and the distance between the focus and the dirctrix.

11. How would you describe the relationship of the non-vertical line through F to the parabolic curve? If you said that this line is the *tangent line* to the curve, you are absolutely right. Let's examine the remarkable reflection property of parabolas that is the key to most of their everyday practical uses. To do this, we want to unhide the vertical line through point D. Click **Display/Show All Hidden**. Everything that was hidden is now showing and is also selected. Deselect the vertical line through D by just clicking on it. Now, click **Display/Hide Objects**, and everything else is hidden away again. Also, click **Graph/Hide Grid.** In order to hide the axis, the origin, and the unit point, select them, and then choose **Display/ Hide Objects.** Also, hide the table and the coordinates of point F. Your picture should now look like the one at right.
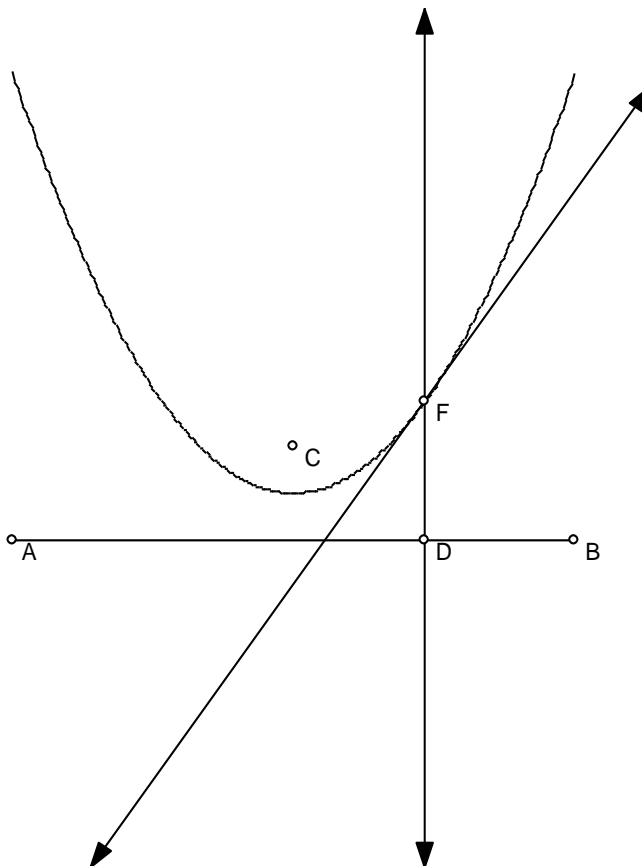
12. Double-click on the tangent line to mark it as a mirror of reflection. Select the vertical line through D, and click **Transform/Reflect**. A new line is constructed, and for better contrast, change its color to red. Drag D back and forth and notice a very important property of this reflected line. It always passes through the *focus* of the parabola! Feel free to drag A, B, or C to different positions. Imagine that the line through D represents a light ray, a sound wave, or anything else. If it is traveling along a path perpendicular to the *directrix* of the parabola, it will reflect off the parabola to its *focus*. It is this important property that is used in satellite dishes, headlights in cars, and in many other applications.

# Computer Lab #30 – Conic Constructions - Ellipses
Part I

1. Start a new sketch, and use the <u>Circle Tool</u> to construct a circle centered at A containing point B. Use the <u>Point Tool</u> to put point C on the circle, and point D inside the circle. Construct segments CD and AC, and also construct the perpendicular bisector of segment CD. Construct the point of intersection F between segment AC and the perpendicular bisector of CD. Your sketch should look like the one at right.
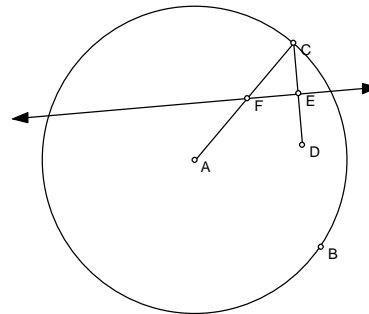
2. Distance FC equals FD by virtue of F being on the perpendicular bisector. Therefore ,
AF + FC = AF + FD = radius of the circle AC. This is true no matter what the position of C as it moves around the circle. Construct segment FD and make appropriate measurements to verify. Select point F, and click **Display/Trace Point**. Slowly drag C around the circle to see a temporary trace of the path of point F.

3. To see a permanent construction of the locus of points, select C and F, and then click **Construct/Locus**. While it is still selected, change its color for a nice contrast. Your sketch should now look like the one at right.
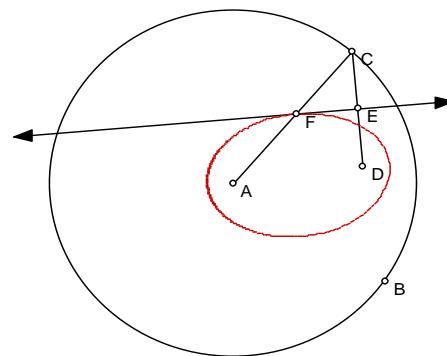
4. The curve constructed is an *ellipse*. Points A and D are the *foci* of the ellipse, and the radius of the circle AC is the length of the major axis. Can you explain why?
Drag on A, B, C, and D to observe the changes in the diagram. Do you see the important property that the perpendicular bisector of CD has? Hopefully you have observed that this line is the tangent line to the ellipse at point F. To see a very nice visual effect, select the tangent line and click **Display/Trace Perpendicular Line**. Now, select point C and the circle and create an Animate button by clicking **Edit/Action Buttons/Animation**. Double-click the Animate button and enjoy the show. Click the button any time to stop the animation.

5. Ellipses have a special reflection property similar to parabolas that involves the foci. To demonstrate this reflection property, hide everything in the sketch but the ellipse, the tangent line, the foci A and D, and points F and C. Change to the <u>Line Tool</u>, and construct line AF. Double-click on the tangent line to mark it as a mirror, and reflect line AF in this mirror using the **Transform** menu. Notice how the reflection always passes through the other focus D as you drag on point C and thus move F around the ellipse.

6. Choose **Graph/Show Grid** and **Graph/Snap Points** to display the grid and turn on the snap to grid points feature. Move points A, B, and C to appropriate lattice points in order to construct the ellipse $9x^2 + 25y^2 = 225$.

Sliders

Before we show another way to draw ellipses, it is worthwhile to spend some time learning how to construct "sliders" in a sketch. Sliders were used in all the predrawn sketches for labs #3 through #8. Now it is time to learn how to construct them for yourself. There is more than one way to construct a slider, but just one method will be used in this section.

1. Start a new sketch. Choose **Graph/Show Grid.** Use the point tool to put three points on the y-axis towards the bottom of the window. Select the three points and the y-axis and choose **Construct/Perpendicular Lines** to construct three horizontal lines through those points. Then, put a floating point on each of those lines. Select the three floating points and choose **Measure/Abscissa(x).** The x-coordinate for each point is displayed. Use the Text Tool to rename each point a,b, and c. Use the Text Tool to rename each measurement a,b, and c as well.
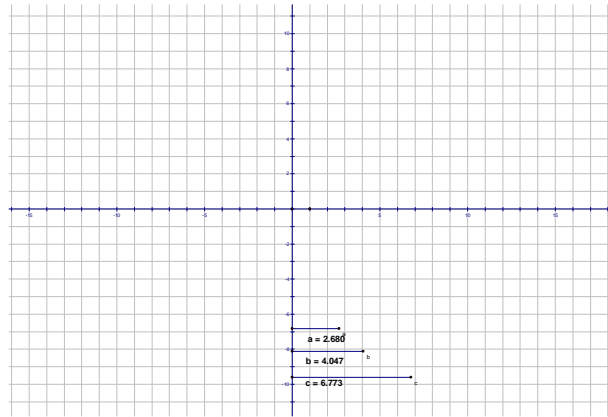
Hide the three lines, and construct segments from the points on the y-axis to the floating points instead. Move the measurements just under the respective slider they are measuring, and the sketch should look like the one at right. Move the sliders back and forth and see how the a,b, and c values change accordingly. Click File/Save, and save this file as *slidertemplate*. Anytime you want to use sliders, just open up this file. If you need less than three sliders, just eliminate, if you need more, just add some more. Three is usually plenty.

To demonstrate the power and importance of sliders, click **Graph/New Function.** Enter in the following into the function window: $ax^3 + bx^2 + cx$. Simply click on the measures for a, b, and c to put those parameters into the equation. Remember you need to use the multiplication symbol * between coefficients and the variable x. Click OK, and the function appears in the sketch window. While it is selected, choose **Graph/Plot Function**, and the graph of the cubic polynomial will appear. Your graph should now look like the one at right.
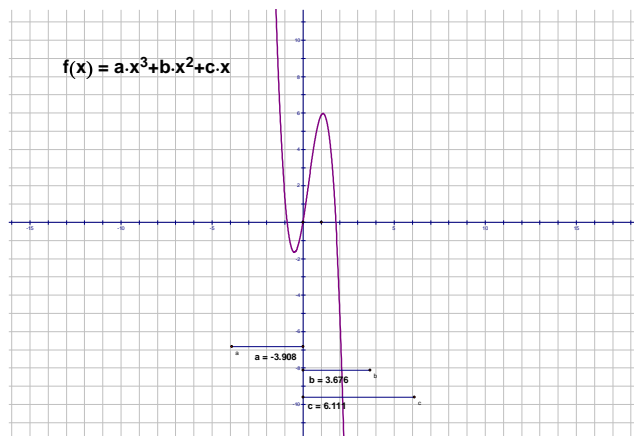
Now, move the sliders to different positions and observe the behavior of the function. This demonstrates the power and usefulness of sliders. Create another family of functions of your own choosing, and investigate.
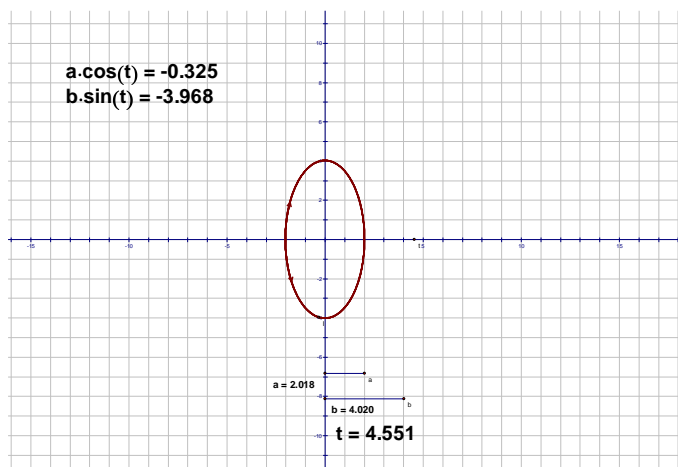
Ellipses Part II – Parametric Equations

1. Open up the *slidertemplate* sketch we just saved. This exercise is only going to use two sliders, so select and hide or delete the c slider from the sketch. Using the point tool, put a floating point on the x-axis and click **Measure/Abscissa(x).** Rename the label for this point t, and its abscissa measurement t as well. One more important preliminary is to go to **Edit/Preferences**, and to change the angle measure units to **radians**.

2. It is now time to type in the parametric equations for an ellipse into the calculator. Click **Measure/Calculate**, and enter in acos(t) and click OK. Then do it again, and this time enter in bsin(t) and click OK. Both acos(t) and bsin(t) should now appear as measures in your sketch.

3. Select acos(t) and bsin(t) in that order, and choose **Graph/Plot As (x,y).** A new point will appear that has the coordinates of the two values you calculated. Select this point and click **Display/Trace Plotted Point**. Move slider a to a value of 2, and slider b to a value of 4, and drag point t on the x-axis back and forth and watch the trace that is produced. To erase the trace, type control-B.

4. To see a more permanent trace of this locus, select point t on the x-axis, and the point being traced, and click **Construct/Locus.** While selected, change the color and line thickness for a nice visual. Your sketch should now look like the one at right.

Turn off the tracer on the point on the ellipse, and explore the changes in the curve when you change the values of a and b.

Some questions:

Construct the ellipse $16x^2 + 9y^2 = 144$ by a and b appropriate values.

What changes would need to be made to the parametric equations in order to make a sketch of an ellipse whose center is not the origin? Try to construct the ellipse $16(x-3)^2 + 9(y+2)^2 = 144$

Challenge. Construct the *foci*, *directrices*, and the tangent line at the point on the ellipse.

Computer Lab #31 – Conic Constructions - Hyperbolas

Problem - Pat and Kim are talking on the telephone during a thunderstorm. After one of the lightning flashes, Pat hears the rumble of thunder *twice* - the first sound coming through the open window, and the second sound coming *through the telephone* ten seconds later. Given that Pat lives two miles east of the center of town, Kim lives two mile west of the center of town, both on the same east-west road, and that sound takes five seconds to travel a mile, draw a map that identifies some of the places where the lightning could have struck. For example, could the lightning have struck the road on which Pat and Kim live? For the purposes of this investigation, assume light and electricity take no time to reach their destination.

1.   Star a new sketch. Click **Graph**/**Plot Points**. When the dialogue box opens, enter the *x* and *y* coordinates appropriately in order to plot the points (–2,0) and (2,0). When you click Done, you should see the two points, along with the origin , the unit point, and the coordinate grid.

2.   Using the Text Tool , change the label for (−2,0) to *Kim*, and also change label for (2,0) to *Pat*. Now, use the Point Tool, and place the cursor on the *x* axis slightly to the right of *Pat*, and click. This will construct a point C that will float back and forth along the *x* axis when dragged. Use the Selection Tool, and select *Pat* and *C* in that order, and click **Construct/Circle By Center** + **Point.** Hide the grid, the origin, the unit point, and the coordinate axis, and your sketch should look like the one at right.

3.   Select *C* and click **Transform/Translate**. Translate by rectangular vector a horizontal component of 2, and a vertical component of 0. The point *C′* is constructed exactly 2 cm to the right of *C*. Construct a segment from *Pat* to *C′*. Now, select point Kim and then this new segment in that order, and click **Construct/Circle By Center** + **Radius**. A circle centered at *Kim* with a radius that is two larger than the circle centered at *Pat* is constructed. Hide the segment and point *C′*. Drag *C* so the circles intersect. Select both circles and click **Construct/Intersections**. Your sketch now should look like the one at right.

Explain why the circle centered at Kim was constructed to be two greater in radius than the other circle.
Explain the significance of the points *of* intersection

4.   Select the points *of* intersection, and click **Measure/Coordinates**. Drag point *C*, and you will see the coordinates for many points that correctly answer the problem at the top of this page. What is the coordinate of the point on the *x* axis (the road on which *Kim* and *Pat* live), where the intersection points *merge*? _____.
**Hyperbolas - Part I (continued)**

6. To see a temporary trace of all these points, select both points and click **Display/Trace Intersections**. Now, slowly drag point C back and forth, keeping *C* to the right of *Pat*. When you stop dragging, the trace of the points remain displayed until you type control-B. If *C* is dragged to the left of (–1, 0), comment about why those points traced out by the intersection points are not possible lightning strike locations.

7. To see a more permanent display of the *locus* of points where the lightning possibly struck, it is easiest to start with a new sketch. Click **File/New Sketch**, and as before, click **Graph/Plot Points** in order to plot the points (–2,0) and (2,0). Label them Kim and Pat respectively again. Select Kim and the origin point (0,0), and click **Construct/Circle by Center + Point**. Drag the unit point to make the picture a little larger, and then hide the grid, the axis, the origin, and the unit point. Your sketch should look like the one at right, which shows a circle of radius two centered at *Kim*, and the point *Pat*.
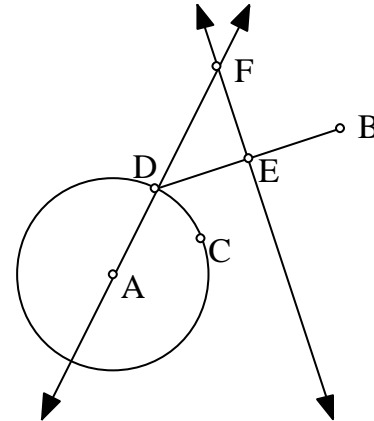
8. Use the Point Tool, and place the cursor on the circle and click. This point *C* will stay on the circle when dragged. Construct the segment from *C* to *Pat*, and then construct the perpendicular bisector of this segment. Change the Segment Tool to the Ray Tool, and construct the ray from Kim to C. Move point C so that the intersection of the ray and the perpendicular bisector of *C to Pat* can be viewed. Construct this intersection point. It is labeled E in the sketch at right. Slowly move *C* around the right half of the circle, and notice that there are positions of *C* for which E does not exist.
Explain why *Kim to E – Pat to E = Kim to E – CE = Kim to C* = 2 no matter what position *C* is in.
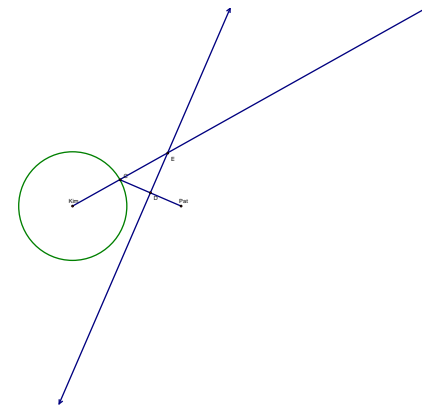
9. Hide the ray, the perpendicular bisector through D, the segment *Pat* to C and point *D*. Select point E, display its coordinates, and click **Display/Trace Intersection**. Once again, slowly drag *C* around the right half of the circle so that *E* stays on the screen. As previously, a temporary trace of the locus of points that make up the possible locations of the lightning strikes is displayed. Explain why the trace of *E* includes exactly the same points that appeared in step 6 at the top of this page.

10. For a permanent tracing of this curve, select points *E* and *C*, and click **Construct/Locus**. While this *locus* is still selected (all the little dark squares are showing on the curve), click **Display/Color/Red**. Select *E* and click **Display/Intersection** in order to turn off the temporary trace feature. Now, drag *C* and watch *E* slide along this curve which is actually one half of a conic section called a *hyperbola*.
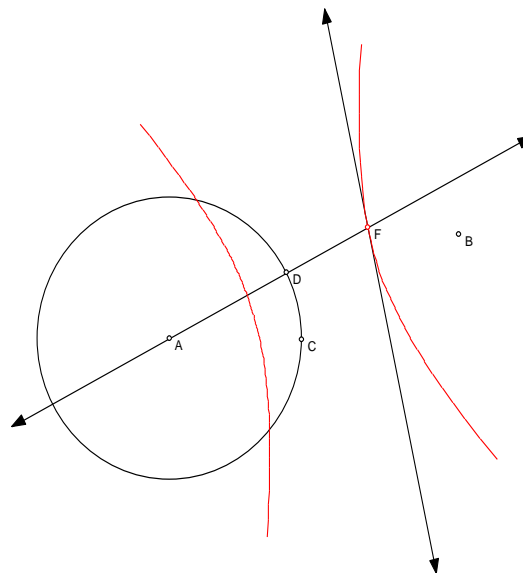
Hyperbolas - Part II

1.   Start a new sketch, and a generalized version of the hyperbola construction will be explored. Using the Point Tool, construct three random points A, B, and C, with C closer to A than B is. Construct a circle centered at A passing through C by selecting A and C and clicking **Construct/Circle by Center + Point**.

2.   Using the Point Tool, place a point D on the circle. Now, construct the segment DB, its midpoint E, and the perpendicular bisector of DB. Change the Segment Tool to the Line Tool, and construct the line AD. Select the two lines, and construct their point of intersection F. Your sketch should look like the one at right. Slowly move D around the right half of the circle and observe the behavior of F.
     Explain why $FA - FB = FA - FD = AD$ = radius of the circle.

What happens to F when D is dragged around to the other side of the circle? Is it still true that $FA - FB = FA - FD = AD$ = radius of the circle? If not, what is true about $FA - FB$? If you are not sure, measure the difference and take note.

3.   To see a temporary trace of this curve, select point F and click **Display/Trace Intersection**. Drag D slowly around the entire circle, and observe that $F's$ locus includes points on two different halves of the curve. Select point F and click **Display/Trace Intersection** to turn off the trace.

4.   Hide segment DB and point E. Select points D and F, and click **Construct/Locus**. While the curve is selected, click **Display/Color/Red** to change its color for some contrast. This red curve is the entire *hyperbola* whose points satisfy the condition that $|FA - FB| = |AD| = |AC|$ = radius of the circle. Drag D slowly around the entire circle and observe that F slides along one branch of the curve or the other. Your sketch should look like the one at right.

**Hyperbolas - Part II (continued)**

5.   Drag any of the original three points to observe how the size and shape of the hyperbola is changed. Be prepared to explain what happened when any of the three points is dragged into a position so that C is no longer closer to A than B is. (i.e. B is inside circle AC). What happens to the hyperbola when this occurs? Have you seen this before?

Explain why this has happened. Think carefully about the roles of *A, B, C*, and *F*.

6.   You were not asked to hide line *EF* this time, as you were in the specific example with Pat and Kim. What important relationship does this line *EF* maintain to the curve at point *F*? This relationship is maintained even when *C* is dragged further from *A* than *B* is.

7.   For contrast, change the color of line *AF* to blue. Now, select *tangent line EF,* and click **Transform/Mark Mirror.** Select line *AF* and click **Transform/Reflect**. The reflected line passes through what other special point? _____. This property of reflection for hyperbolas is analogous to the reflection property for ellipses. In fact, drag point *C* so that *C* is further away from *A* than *B* is, and observe what happens. Drag C so that the figure is a hyperbola again.

8.   Hide everything in your sketch other than the hyperbola and its focal points *A* and *B,* and point *C*. Using the Point Tool put the cursor on the hyperbola and click. This point *G* is now a point that will stay on the curve when dragged. Try it. Construct lines *AG* and *BG*. Construct the angle bisector of angle *AGB* by selecting the three points in that order and clicking **Construct/Angle Bisector**. A ray through *G* is constructed. To construct a line instead, put a point *H* on the ray through *G* and construct the line *GH*. Hide the lines *AG*, *BG*, and point *H*, and drag point *G* around. This line is the tangent line for the hyperbola at point *G*.
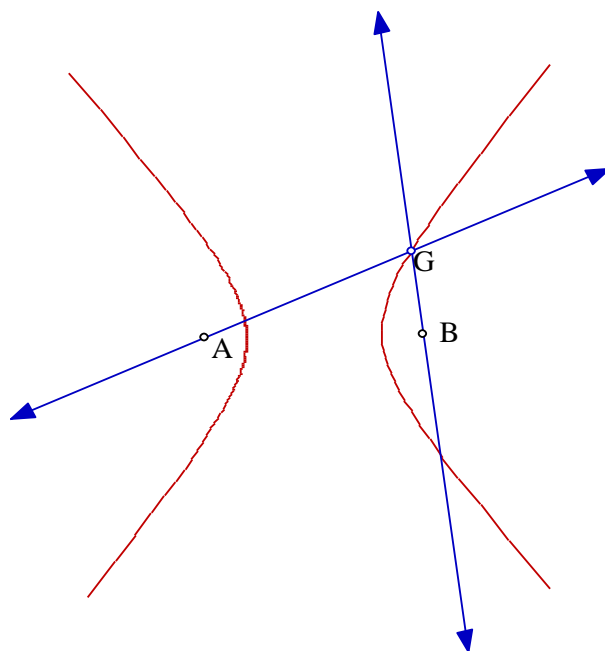
Computer Lab #32 Trigonometric Applications -

A Trigonometric Curve Slider Lab for $y = a\cos(bx) + c$

1. Open up the sketch called *slidertemplate*. Using the Text Tool to double-click on each slider value, change the precision for each to Tenths. Make a = 1, b = 2, and c = 0.

2. Click Graph/New Function, and type in $a*\cos\ b*x\ +c$ and hit OK. a, b, and c all come from values on display in the sketch. When the new function is displayed, choose **Graph/Plot Function.** Respond yes to changing units to radians, and the cosine curve for the current values of parameters a,b, and c will be graphed. Your sketch should look like the one at right.

g(x) = a·cos(b·x)+c

a = 1.0
b = 2.0
c = 0.0
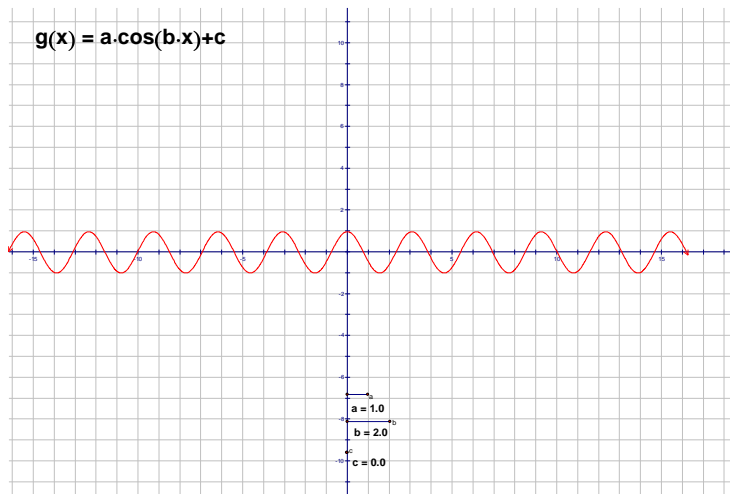
3. Set c = 0, a = 1, and b = 1, and the curve $y = \cos(x)$ should be displayed. Keep c at 0, and move the a slider-point back and forth slowly in order to notice the effect on the curve. Summarize the role a plays in the equation $y = a\cos(x)$.

4. Set c = 0 and a = 2, and move the b slider-point back and forth slowly in order to notice the effect on the curve. Summarize the role b plays in the equation $y = 2\cos(bx)$.

5. Set a = 2 and b = 1, and move the c slider-point back and forth slowly in order to notice the effect on the curve. Summarize the role c plays in the equation $y = 2\cos(x) + c$.

6. Write the equation of the following curves. Keep in mind that the precision shown for the slider values are rounded to the nearest tenth. Move the sliders to appropriate values to draw a sketch of the curve as you a check on your work.

a) There is a crest at (0,2) and an adjacent valley at (1,–2).

b) There is a valley at (0,–3) and an adjacent crest at (4,3).

c) There is a crest at (0,5) and an adjacent valley at (2,–3).

d) There is a valley at (0,–2) and an adjacent crest at (6,0).

Computer Lab #33 -Time Minimization Investigation and Snell's Law

Problem - David is at the point (0,8) offshore, needing to reach the destination E (12,–3) on land as quickly as possible. The shore of this lake is the x axis. David is in a boat that travels at 10 uph, with a motor bike on board that will do 20 uph once the boat reaches land. The problem is to find the landing point F = (x,0) that will minimize the total travel time from David to E.
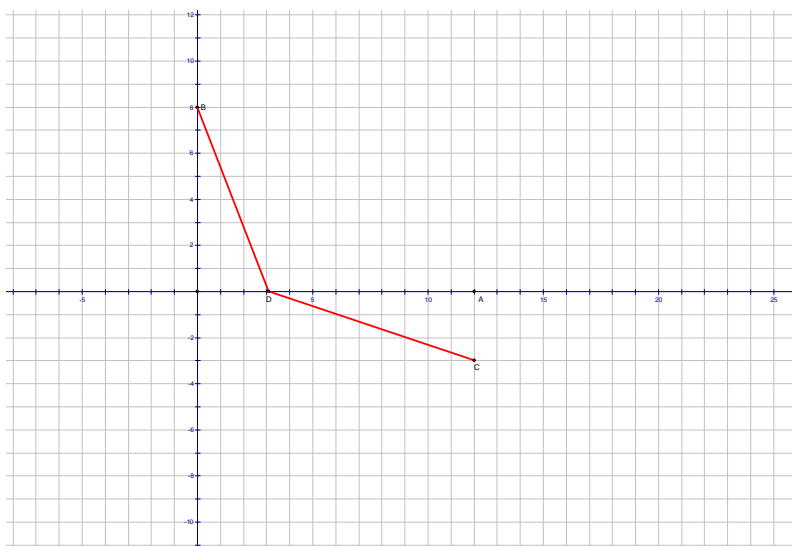
1. Click **Edit/Preferences/Units**, and make all precision amounts in *hundred thousandths*, and distance unit cm. Click **Graph/Plot Points**, and plot the points A = (12,0), B = (0,8), and C = (12,–3). Hide the unit point . Use the <u>Point Tool</u>, to put a floating point D on the x axis. Now use the <u>Segment Tool</u> to construct segments *DB* and *CD*. Your sketch should now look like the one above.

2. Select segments *DB* and *CD* and click **Measure/Length**. Click **Measure/Calculate**... to open up a calculator. Enter the expression *DB*/10 + *CD*/20 in the calculator. (Click the measurements to put them in the calculator window.) Click **OK**. Although the units are showing cm, we know that it is really time that this measurement represents. What should the units be?

3. Use the <u>Selection Tool</u> to drag *D* back and forth between *A* and the origin, trying to make the displayed measurement as *small* as possible. When D is in the optimal position, measure the distance *A* to *D* by selecting the two points and clicking **Measure/Distance**. Record the two values displayed in the sketch.
AD = _____   *DB*/10 + *CD*/20 = _____

4. If the objective had been to make the sum *DF*/10 + *EF*/10 as small as possible, your search would not have led you to the same point *D*. Guess the approximate location of point D that would have been optimal, and then check your guess by doing the search. You will need to reopen the calculator, and enter the expression *DB*/10 + *CD*/10. Record your findings.

   AD = _____   *DB*/10 + *CD*/10 = _____

5. New example: Search for the point *F* that makes *DB*/20 + *CD*/10 as small as possible.

   AF = _____   *DB*/20 + *CD*/10 = _____

   Notice that segment *AD* is longer in the last example than the two preceding examples. Explain why this could have been predicted.
 Time minimization investigation (continued)

6. Select the origin and points B and D in that order, and click **Measure/Angle**. Once used in a measurement, the origin is given a label and is now called point E. Now measure angle *ACD*. As *D* moves between *E* and *A*, these angles change. Explain how you might predict which angle would be larger for an optimal location for *D* in any given problem of the form: minimize *DB/ r1 + CD/ r2*. Will the angles ever be the same?

7. You have explored three problems of this same general form: Find *D* so as to minimize the expression *DB/ r1 + CD/ r2*. Remember, this expression represents *time*, because the numerators represent *distances* and the denominators represent *rates*.

8. A few more examples and measurements are needed to reveal a pattern. Fill in the missing entries of the table below, and notice that there are two new quantities of interest. Each row represents a separate problem and thus a separate search.

R1	R2	Origin(E) to D	*DB/ r1 + CD/ r2*	ED/BD	DA/DC
10	30				
10	20				
10	10				
40	30				
20	10				
30	40				
30	10				

9. As rate *r2* increases, what happens to the optimal position of *D*? What happens to the length of segment *DC*? What happens to the size of angle *ACD*? Are *r2* and angle *ACD* linearly related?

10. The simple pattern known as **Snell's Law** relates *r1*, *r2*, ED/BD, and DA/DC. It can be inferred by examining the table closely.

     Express this relationship in words.

     Express this relationship in symbols.

11. Write a trigonometric description of Snell's Law that relates the rates *r1* and *r2* to the angles *EBD* and *ACD*.

Computer Lab #34 - Problem Solving Through Modeling

1. A line with positive slope passes through the point (-2,4) and thus forms a right triangle in the 2nd quadrant with the coordinate axis. Find the slope that will minimize the area of this triangle. The key here is to construct a floating point that will allow you to easily change the slope of the line and arrive at the optimal solution. Extend the problem by repeating the problem using a few different points (a,b) and see if you can find a pattern that emerges for the minimal area slope.
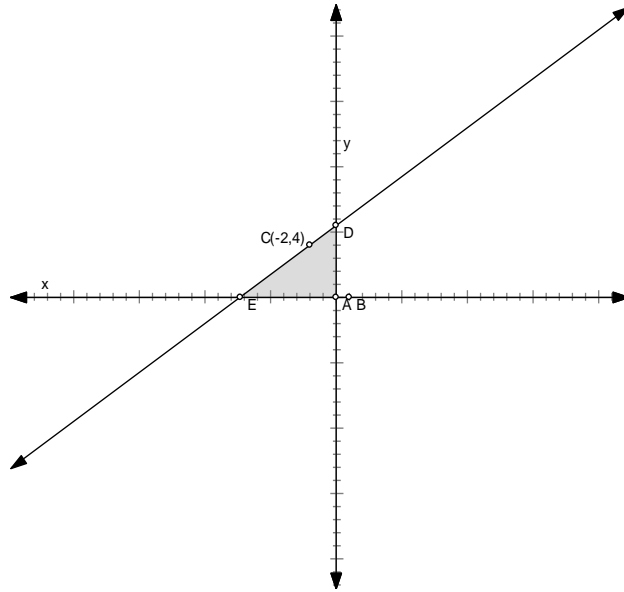
2. A rectangular piece of paper is 12 cm wide and 10 cm high. It is folded so that corner D is matched with a point F on edge BC. (EG is the fold line). Find the length of DE that maximizes the area of triangle EFC.
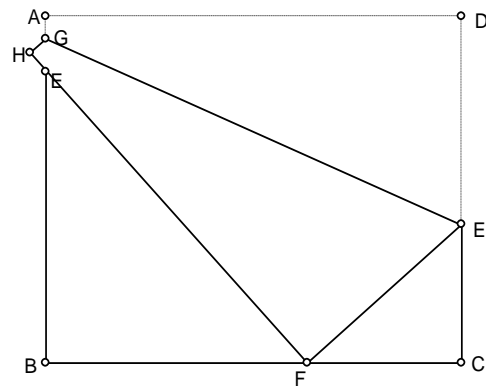
3. A fence of height 8 ft is 6 ft from the side of a building. A ladder, whose foot is outside the fence, leans against the side of the building and makes a 63 degree inclination with the ground. The ladder barely touches the top of the fence. Find the height of the ladder.
.
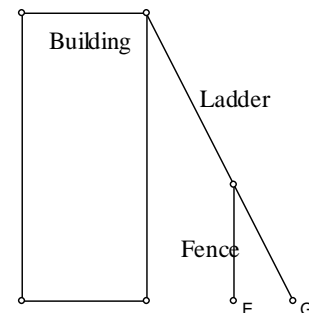
4. A section of a Gothic window looks like the figure below. It is framed by two circular arcs and segment AB. The center of arc BC is A, and the center of arc AC is B. The complete circle shown is tangent to both arcs and to segment AB. Given that AB = 8, find the radius of the circle.
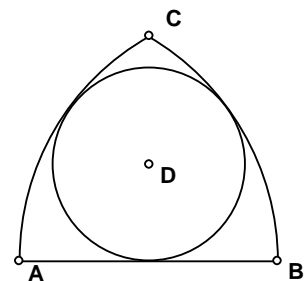
**Problem Solving Through Modeling - (continued)**

5. A circular Harkness table is placed in a corner of a room so that it touches both walls. A mark is made on the edge of the table, exactly 8 inches from one wall and 9 inches from the other. What is the diameter of the table? The key to this model is to put a floating center on the line y = x and a circle passing through the fixed point (8,9). Surprisingly, there are two answers to this problem and the are both integers. Good Luck finishing the problem.
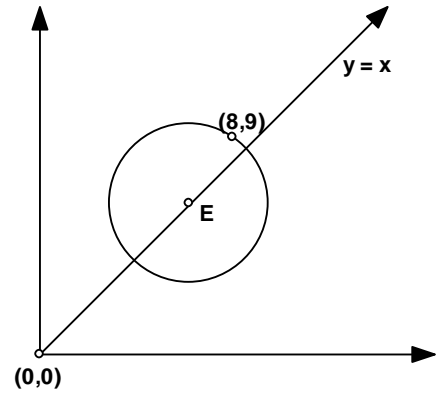

6. The screen at a movie theater is 14 feet tall, and has its bottom edge 8 feet above eye level. If Bob is sitting 44 feet from the front wall it is not satisfactory, for the screen is far away and only subtends a 16.3 degree viewing angle. Sitting 4 feet away from the front wall is just as bad. The subtended viewing angle is the same 16.3 degrees. Verify these facts. Find the optimal viewing distance by finding the distance from the front wall that maximizes the subtended viewing angle x (i.e. makes the screen seem the largest.)
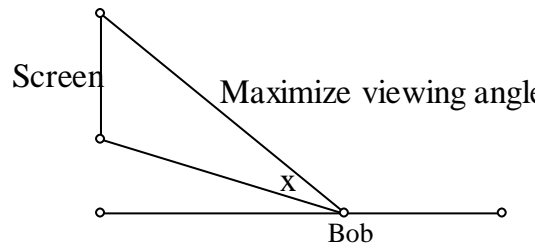
7. The sides of an isosceles trapezoid are 8, 10, 20 , 10. What is the area of the trapezoid? How far must the non-parallel sides be extended to make them meet?
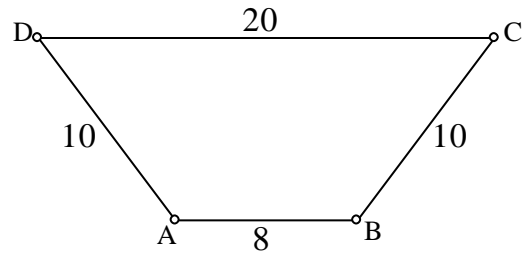

8. Build a sketchpad model that will model the location of the minute and hour hands of an ordinary analog clock. Use this model to determine the angle formed between the minute hand and hour hand at 8:20.
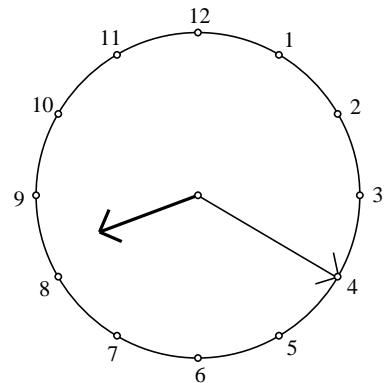
Advanced: Construct an animation so that when the animation is activated, the clocks hands will move as they should in a real clock.

# Computer Lab #35 - Miscellaneous Construction Activities

1. Given a random triangle, construct 3 circles, each one centered at a different vertex, such that the circles are mutually tangent.
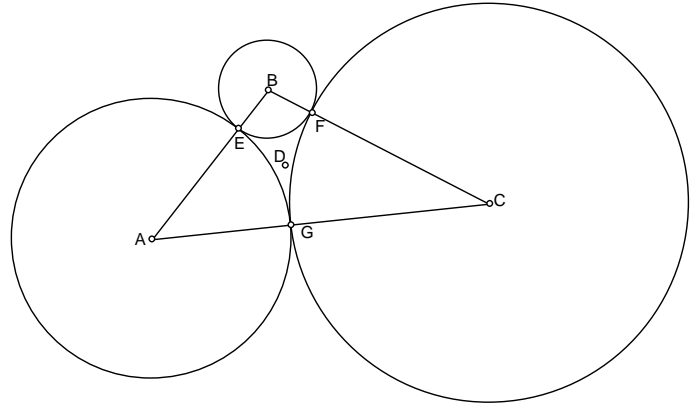
2. The figure at right is called an *arbelos.* It is formed by three semicircular arcs, which have diameters AB, BC, and AC. Segment BD is perpendicular to AB, and ABC is straight. Given that BD has length 8 inches, find the **area** enclosed by the arbelos. (The unshaded area.) It is not a misprint or mistake that the only length given is BD.
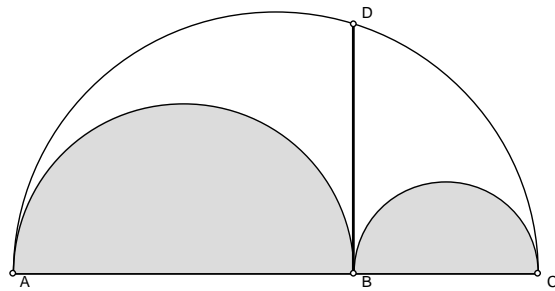
3. Construct a circle tangent to two parallel lines and a transversal intersecting them.
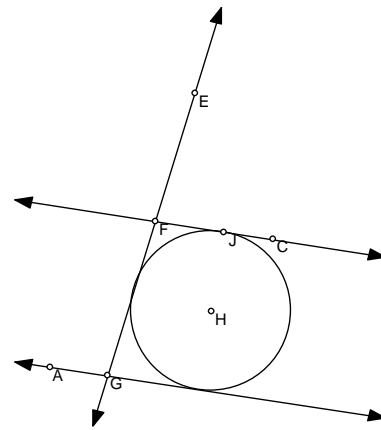
4. The curve below that is traced out by a point on a rolling circle is called a *cycloid*. Make a GSP construction like the one below. If possible, make it possible for the circle to be animated as it rolls along the line.
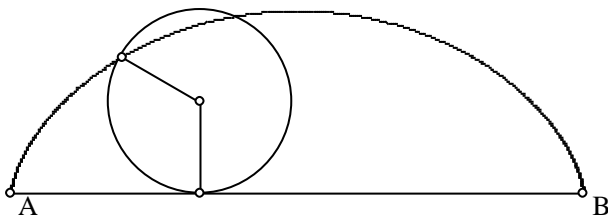
Computer Lab #36 - Iteration and Fractals

Iteration in GSP constructs iterated images of a set of related geometric objects according to an iteration rule that you define. This command is available when you select some combination of pre-image points that must be independent that define other points (image points) in your sketch. It is best illustrated with examples, so two will follow.

1. Start a new sketch and construct a triangle ABC and the midpoints of the three sides D,E, and F. Now connect D,E, and F to form another triangle.

2. Triangle DEF is called the *midpoint triangle* of ABC. This process of constructing midpoint triangles could be repeated or iterated by specifying an iteration rule in GSP. You do this by defining a map that identifies the image for each pre-image. $A \Rightarrow D, B \Rightarrow E, and\ C \Rightarrow F$.

3. Select A,B, and C and choose **Transform/Iterate**. A dialogue box opens up that will allow you to define the mapping. All mappings are done point to point, but GSP looks at your sketch and "connects the dots" so to speak to form a complete image. You enter in the mapping by clicking on the matching image point in the sketch itself. In order to match D to A, click on point D in the sketch. GSP produces a set of iterated images for each object affected by your mapping. Under the dialogue box **Display** menu, you can increase or decrease the number of iterations that are shown. Under the dialogue box **Structure** menu you can add another mapping for more complicated iterations. We will do this next. For now, simply map A to D, B to E, and C to F, and click iterate. The sketch for 3 iterations should appear and look like the one at right.

Multiple Mappings

To construct the fractal known as *Sierpinski's gasket* more mappings need to be defined than just the one in the previous example. Let's illustrate again with an example.

1. Start a new sketch and construct a random triangle ABC and its midpoints D,E, and F as illustrated to the right. Do not connect D, E, and F this time.

2. Select A,B, and C and choose **Transform/Iterate**. This time you are going to do three different mappings. After assigning your first mapping, you use the Structure menu to add another map. The mappings you will use are at right.

$map1\ A \Rightarrow F, B \Rightarrow B, C \Rightarrow D$

$map2\ A \Rightarrow E, B \Rightarrow D, C \Rightarrow C$

$map3\ A \Rightarrow A, B \Rightarrow F, C \Rightarrow E$

3. After doing the three mappings and clicking on iterate, your 3 iteration sketch should look like the one at right. You can select the entire triangle and use the + and – keys on the keyboard to increase and decrease the number of iterations in your sketch. Try it.

4. Experiment with iteration and triangle interiors, and and see if you can sketch the diagram shown below.
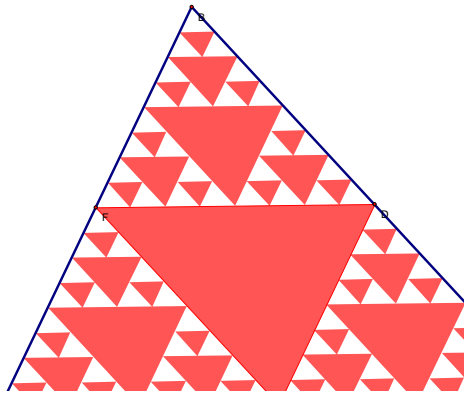
5. Below is a picture of another fractal created using four iteration depth in Sketchpad. See if you can create it.
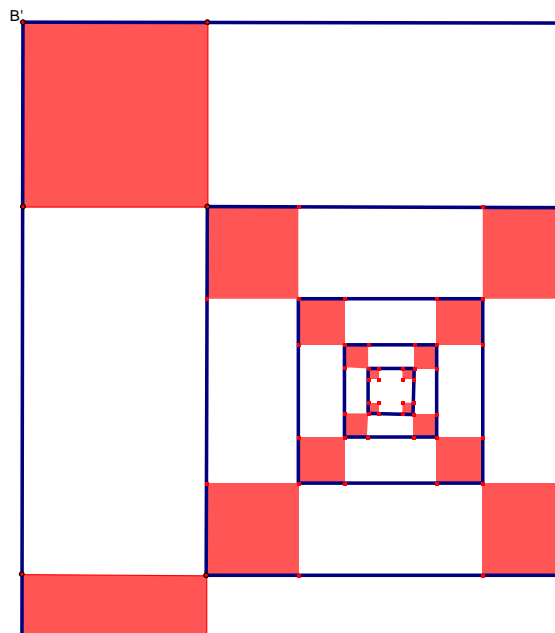
Computer Lab #37 – Rings Lab on Direct Variation

1. Start a new sketch and choose **Edit/Preferences/Units** and change the precisions to thousandths. Now, use the Segment Tool to put a random line segment AB that is a few inches long in the middle of the sketch window. While it is selected, click on **Measure/Length.** After selecting just the segment AB again, choose **Construct/Midpoint** to construct the midpoint C of AB.

2. Select C and then either A or B, and choose **Construct/Circle By Center+Point** in order to construct a circle with AB as the diameter. While it is still selected, choose **Measure/Circumference**.
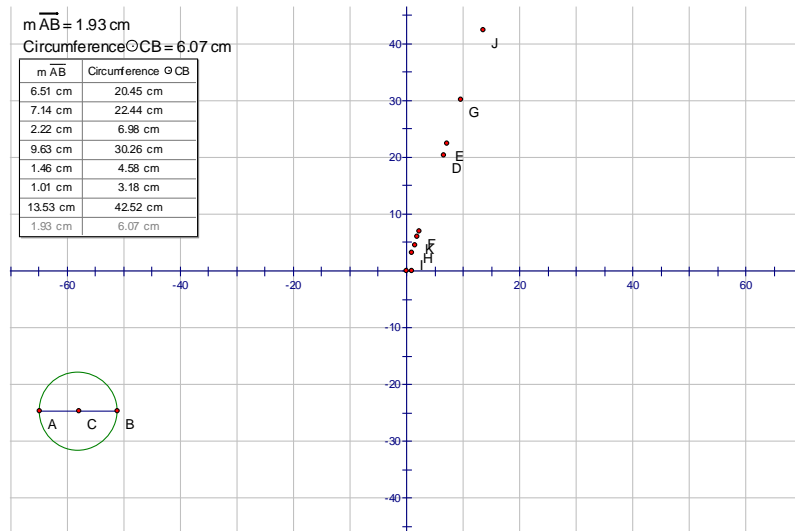
3. Select the measure of length AB first and then the measure of the circumference second, and then choose **Graph/Tabulate**. A table should appear that has the length of AB as the header for one column and the circumference as the header for the other. Also, the current value of each measure will appear in the first row of the table. Double-click on the table to permanently put those vales in the table. Your sketch should now look like the one at right.

m $\overline{AB}$ = 5.64 cm
Circumference⊙CB = 17.71 cm

m $\overline{AB}$	Circumference ⊙ CB
5.64 cm	17.71 cm
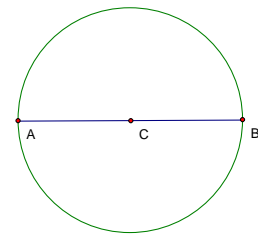
4. Drag either point A or B to change diameter and circumference of the circle. Doubleclick on the table, and the new values for the length of AB and the circumference of the circle will be added to the table in a new entry. Repeat this process at least seven more times of changing the diameter of the circle to make it both bigger and smaller, and then double-clicking on the table each time to add data to the table. When finished, make the circle very small and drag it to the bottom left of the sketch window. Then, select the table and choose **Graph/Plot Table Data.** Click Plot when the dialogue box appears. Your sketch should now look like the one at right.

m $\overline{AB}$ = 1.93 cm
Circumference⊙CB = 6.07 cm

m $\overline{AB}$	Circumference ⊙ CB
6.51 cm	20.45 cm
7.14 cm	22.44 cm
2.22 cm	6.98 cm
9.63 cm	30.26 cm
1.46 cm	4.58 cm
1.01 cm	3.18 cm
13.53 cm	42.52 cm
1.93 cm	6.07 cm

5. Notice that GSP has automatically chosen a scale that fits the data on the screen. The origin and the unit point (1,0) automatically appear, and should be hidden since they are not part of the data set. Select them and choose **Display/Hide Points**. It also makes sense not to see the labels of all the data points. Use the <u>Text Tool</u> to hide any labels for the data points. Choose any two points in your data set (choosing two that are far apart is better), and then construct the line containing those two points. Choose **Measure/Equation**. This equation describes the relationship between what two quantities?

6. Use the equation to predict the circumference of circles with diameters of: a) 2   b) 4   c) 6

7. Does circumference vary directly with the diameter? Explain.

8. What role does the slope of the line play, and where have you seen this slope number before?

9. Select the *y*-axis only and choose **Edit/Cut**. Notice that the entire grid and all the plotted points that depended on the grid are deleted. Also delete the table and the circumference measure. The only objects remaining in your sketch should be the circle and the diameter measure. Now, select the circle and measure its Area. Select the diameter and the area in that order, and choose **Graph/Tabulate** to now start a table with diameter and area similar to our last table of diameter and circumference.

10. You are now going to repeat everything from step #4 in which you change the size of the circle and double-click on the table to collect data points. After collecting at least seven or more data points, choose **Graph/Plot Table Data** and click Plot when the dialogue box appears. A new scatter plot appears of diameter versus area.

11. You should notice that this data is clearly NOT linear. Try and determine the function that does relate area and diameter. Once you think you have the right function, choose **Graph/New Function**, enter in the function, and then choose **Graph/Plot Function** to see if it goes through your data points.

Computer Lab #38 – Balance Model for Solving Linear Equations

1. An equation such as $5m+2c=4c+m$ can easily be modeled by a balance diagram similar to the one shown at right that shows a cross section of a balance with five marbles and two cubes on the left side perfectly balancing four cubes and one marble on the right side..

2. Open the file **balancedemo.gsp** and the balance diagram shown above appears in the sketch window. Click on the action box to the right of each step number to see a demonstration of how to use the balance model to solve the given equation.

3. Open the file **equations.gsp** and a new balance model like the one shown at right is now in the sketch window. It should have the title Equation 1. First, write a mathematical equation that is equivalent to the balance problem shown. Use the Selection Tool (the arrow tool) in sketchpad to select and subtract off the balance the same number of cubes and marbles from each side. Write the equivalent mathematical steps on this worksheet. Multiply both sides by the appropriate fraction in order to determine the number of marbles equivalent to one cube or vice versa. Once finished solving the equation on this worksheet, click on the Show Solution button to check your answer. Below is an example of what should appear on your paper for Equation 1.

$$3m+5c=9m+3c$$
$$-3m \qquad -3m$$
$$\underline{\qquad -3c \qquad -3c \qquad}$$
$$2c \; = \; 6m$$
$$c \; = \; 3m$$

4. Once finished with Equation 1, click on the next tab in the lower left hand corner of the sketch labeled Equation 2 to solve the next balance equation shown at right. Notice in this equation, you will be finding the number of cubes that balances one marble, or solving the equation for *m* in terms of *c*. Use the space below to show work.

5. Click on the tab in the lower left marked Equation 3 and solve below. After solving, check your answer by clicking on Show Solution in the sketchpad diagram.

6. Click on the tab in the lower left marked Equation 4 and solve below. After solving, check your answer by clicking on Show Solution in the sketchpad diagram. Notice that this is the first example where the solution is not an integer equivalency. This is perfectly OK as far as the algebra is concerned, but the balance model stops one step short of the depicting the final solution. Why?

7. Click on the tab in the lower left marked
   Equation 5 and solve below. After solving,
   check your answer by clicking on Show
   Solution in the sketchpad diagram. Notice
   that this is another example where the solution is not an integer equivalency and also you will be
   solving for *m* in terms of *c*.

8. To sole the equation $5c - 2m = 3c + 6m$ for *c*, a balance model cannot be used because the left side
   contains a $-2m$. However, the algebra solution process is almost exactly the same. By adding $2m$
   to both sides and subtracting $3c$ from both sides, the equation $2c = 4m$ remains. Taking one half of
   both sides yields the final answer $c = 2m$. Solve the following equations for *c* in terms of *m*
   without the aid of a balance model.

   a) $3c - 5m = m + c$

   b) $6m - 3c = 2m + 2c$

   c) $4c + 3m = 8c - 2m$

   d) $5.5c - 2.2m = 1.5c + 1.8m$

Notes about Using Computers in a Lab Situation

- Worksheet is pre-tested

- Do not make it too ambitious, especially in the beginning. Allow for plenty of time.

- Make sure computer lab is checked out so that both hardware and software is working properly

- The activity should be relevant to what is being studied in class.

- Make sure there is something to collect. Often teams of two work well.

- Let them know that something related will be on the next test.

- Have some ideas for the faster students who finish first. This could include helping the others.

- Make sure you discuss and summarize the results when you hand back the worksheets.